Phenomenological formula of total reaction cross sections for low-energy systems

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This Brief Report presents a phenomenological formula of total reaction cross sections for different reaction systems at low energies, taking into account the case of weakly bound nuclei as projectiles. To get this phenomenological formula, a large set of experimental data were collected and compared. Based on the experimental total reaction cross sections, three parameters were used to modify Wong's formula. The total reaction cross sections of different systems at low energies including weakly bound projectiles were reproduced using the modified formula.

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The total reaction cross section is part of the basic information in a nuclear reaction and is important for a variety of applications in the areas of astrophysics, nuclear energy, and national security [1]. It has been studied both in theory and in experiment for a long time. The elastic scattering cross sections of a large number of reaction systems have been measured and the total reaction cross sections have been extracted, using the optical model [2]. In addition, there are some direct measurements of the total reaction cross sections [3]. Recently, the development of radioactive ion-beam facilities has stimulated a great interest in studying the properties of nuclei away from the valley of stability and more and more experimental data about weakly bound nuclei on stable targets have been attained. Because of the lower binding energy of weakly bound nuclei, direct reaction channels have to be taken into account in the total reaction cross section measurement. Furthermore, many reaction cross sections of unstable nuclei are difficult to measure directly, and therefore theoretical calculations are needed. Most of the theoretical methods focus on the calculation of the reaction cross sections of tightly bound nuclei in the intermediate- to high-energy range [4,5].

During the past few years, several reduction methods were proposed to compare the total reaction cross sections in different nuclear reaction systems [6,7]. One of these methods is based on Wong's formula [8], which is a model to calculate the fusion cross section for different reaction systems at low energies. In this Brief Report, a large set of the experimental data was analyzed to obtain a phenomenological formula for the total reaction cross sections for reactions with both tightly and weakly bound projectiles at low energies.

To analyze different kinds of systems together, different reduction methods were introduced. Shorto *et al.* [7] suggested using the total reaction function to evaluate the total reaction cross sections of different kinds of systems. The total reaction function, F_{TR}, is dimensionless. It is associated with the total reaction cross section σ_{TR} and the dimensionless variable x which relates to the collision energy. F_{TR} and x are defined as

$$F_{\mathrm{TR}}(x) = \frac{2E}{\hbar\omega R_B^2}\sigma_{\mathrm{TR}} \quad \text{and} \quad x = \frac{E - V_B}{\hbar\omega}, \tag{1}$$

where R_B, V_B, and $\hbar\omega$, which can be extracted from the São Paulo potential (SPP) [9], are the parameters associated with the barrier radius, height, and curvature, respectively, and E is the kinetic energy in the center-of-mass frame. Typical values of these parameters are given in Table I. In Ref. [10] the system-independent fusion function $F_0(x)$ is defined as

$$F_0(x) = \ln[1 + \exp(2\pi x)], \tag{2}$$

called the universal fusion function (UFF), which comes from Wong's formula [8],

$$\sigma_F \simeq \sigma_F^W(x) = R_B^2 \frac{\hbar\omega}{2E} \ln\left[1 + \exp\left(\frac{2i(E - V_B)}{\hbar\omega}\right)\right]. \tag{3}$$

Wong's formula works well in many cases. However, it cannot reproduce the results of optical model calculations for light systems at sub-barrier energies; moreover, it breaks down when the breakup coupling is important. The reduction method can be used to compare different kinds of systems directly. In the present work, a large number of detailed works of tightly bound projectile systems are selected from the experiment. Depending on the Coulomb barrier, reaction systems were separated into three regions; $V_B < 10$ MeV, $10 < V_B < 20$ MeV, and $V_B > 20$ MeV.

First, the collision systems with tightly bound projectiles in the $V_B < 10$ MeV system are studied. Figure 1(a) shows reaction functions for the collisions of several systems. The systems of tightly bound projectiles are $^{12}C + ^{11}B$ [11], $^{12}C + ^{12}C$ [12,13], $^{16}O + ^{12}C$ [14–16], and $^{16}O + ^{16}O$ [17]. The reaction functions of these systems are given in Fig. 1(a) with UFF as a reference. It can be seen from Fig. 1(a) that the reduced total reaction functions are under the UFF. To reproduce the experimental data, three parameters were added to the UFF,

$$F_{\mathrm{tot}}(x) = IM \ln\{1 + \exp[2\pi(x + P)]\}, \tag{4}$$

where I, M, and P are dimensionless parameters.

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TABLE I. Typical values of R_B, V_B, and $\hbar\omega$ used in this work. The values are from SPP [9]. One should note that these parameters slightly depend on the collision energy. For $^{12}C + ^{12}C$, parameters are given in three different energies.

Systems	E (MeV)	R_B (fm)	V_B (MeV)	$\hbar\omega$ (MeV)	Ref.	Systems	E (MeV)	R_B (fm)	V_B (MeV)	$\hbar\omega$ (MeV)	Ref.
$^{12}C + ^{11}B$	344.5	7.84	5.02	2.43	[11]	$^{6}He + ^{27}Al$	12	8.6	4	2.3	[18]
$^{12}C + ^{12}C$	180	7.9	5.99	2.59	[12,13]	$^{6}He + ^{58}Ni$	9	9.35	7.97	2.93	[18]
$^{12}C + ^{12}C$	300	7.83	6.04	2.59	[12,13]	$^{6}He + ^{65}Cu$	22.6	9.55	8.11	2.89	[18]
$^{12}C + ^{12}C$	360	7.79	6.06	2.61	[12,13]	$^{6}Li + ^{27}Al$	10	8.3	6.22	2.8	[19]
$^{16}O + ^{12}C$	62	8.14	7.77	2.71	[14–16]	$^{7}Li + ^{27}Al$	18	8.45	6.11	2.64	[19]
$^{16}O + ^{16}O$	75	8.28	10.2	2.84	[17]	$^{9}Be + ^{27}Al$	32	8.5	8.09	2.74	[19]
$^{12}C + ^{28}Si$	65	8.51	13.05	3.08	[20,21]	$^{6}Li + ^{58}Ni$	14	9	12.36	3.68	[22]
$^{12}C + ^{40}Ca$	180	8.75	18.15	3.39	[13]	$^{6}Li + ^{59}Co$	29.6	9.05	11.87	3.59	[23]
$^{13}C + ^{28}Si$	60	8.6	12.09	3.01	[20]	$^{6}Li + ^{64}Zn$	22	9.16	13.07	3.67	[24,25]
$^{16}O + ^{28}Si$	75	8.66	17.11	3.17	[20]	$^{7}Be + ^{58}Ni$	21.4	8.95	16.6	3.9	[22]
$^{12}C + ^{58}Ni$	344.5	9.08	24.55	3.68	[11]	$^{7}Li + ^{64}Zn$	20	9.34	12.84	3.38	[25]
$^{12}C + ^{197}Au$	344.5	11.2	56.92	4.65	[11]	$^{6}He + ^{120}Sn$	17.4	10.5	12.78	3.41	[26]
$^{12}C + ^{90}Zr$	120	9.87	32.47	3.96	[11,13]	$^{6}Li + ^{112}Sn$	25	10	20.06	4.25	[29]
$^{16}O + ^{58}Ni$	40	9.42	31.65	3.69	[27]	$^{6}Li + ^{116}Sn$	26	10.1	19.9	4.17	[29]
$^{16}O + ^{60}Ni$	56	9.48	31.46	3.66	[27]	$^{6}Li + ^{208}Pb$	35	11.25	29.44	4.79	[30]
$^{16}O + ^{62}Ni$	70	9.54	31.28	3.63	[27]	$^{6}Li + ^{209}Bi$	36.9	11.25	29.79	4.83	[30]
$^{16}O + ^{64}Ni$	70	9.61	31.08	3.59	[27]	$^{8}B + ^{58}Ni$	25.3	8.9	20.8	4.15	[22]
$^{12}C + ^{208}Pb$	96	11.48	57.74	4.7	[13,28]	$^{6}He + ^{197}Au$	27	11.5	18.6	3.76	[18]
$^{16}O + ^{208}Pb$	129.5	11.63	76.09	4.63	[28]	$^{6}He + ^{208}Pb$	27	11.6	19.11	3.86	[18]

The same region as that of the tightly bound projectile systems is made for weakly bound projectile systems. Figure 1(b) shows the reaction functions for collision systems with weakly bound projectiles in the $V_B < 10$ MeV system: $^{6}He + ^{27}Al$, $^{6}He + ^{58}Ni$, and $^{6}He + ^{65}Cu$ [18]; $^{6}Li + ^{27}Al$, $^{7}Li + ^{27}Al$, and $^{9}Be + ^{27}Al$ [19]. Equation (4) is applied to fit all these points to obtain the values of three parameters. The results are given in Table II. There are no distinct differences between weakly bound and halo projectiles in this barrier region.

Then systems in the second region, $10 < V_B < 20$ MeV, were investigated. Figure 1(c) shows the results of several tightly bound projectile reactions: $^{12}C + ^{28}Si$ [20,21], $^{12}C + ^{40}Ca$ [13], $^{13}C + ^{28}Si$ [20], and $^{16}O + ^{28}Si$ [20]. For the reactions in this region, the reduced total reaction cross section at the same energy is larger than that of the system in Fig. 1(a). All these experimental points are reproduced well by the UFF. The improved UFF was applied to obtain the best-fit parameters. Figure 1(d) shows the reaction functions for collision systems with weakly bound projectiles in the $10 < V_B < 20$ MeV range: $^{6}Li + ^{58}Ni$ [22], $^{6}Li + ^{59}Co$ [23], $^{6}Li + ^{64}Zn$ [24,25], $^{7}Be + ^{58}Ni$ [22], $^{7}Li + ^{64}Zn$ [25], and $^{6}He + ^{120}Sn$ [26]. For the reactions in this region, the halo projectiles and weakly bound projectiles show some differences. The reduced total reaction cross section of the halo nucleus (^{6}He) is slightly larger than that of the weakly bound nucleus. Therefore, the improved UFF is applied to fit the experimental data of weakly bound projectiles and halo projectiles separately, even though the data of ^{6}He only include four points. The values of the three parameters are given in Table II. To confirm the parameter values for the halo projectiles, more experimental data are needed.

Figure 1(e) shows the reaction functions for collision systems with tightly bound projectiles in the $V_B > 20$ MeV range: $^{12}C + ^{58}Ni$ and $^{12}C + ^{197}Au$ [11], $^{12}C + ^{90}Zr$ [11,13], $^{16}O + ^{58,60,62,64}Ni$ [27], $^{12}C + ^{208}Pb$ [13,28], and $^{16}O + ^{208}Pb$ [28]. The behavior is similar to those in Fig. 1(c).

Finally, collision systems with several weakly bound projectiles in the $V_B > 20$ MeV range were studied: $^{6}Li + ^{112,116}Sn$ [29], $^{6}Li + ^{208}Pb$ and $^{6}Li + ^{209}Bi$ [30], $^{8}B + ^{58}Ni$ [22], $^{6}He + ^{197}Au$, and $^{6}He + ^{208}Pb$ [18]. The reduced total reaction functions show a distinct deviation from the UFF although they have a slightly broader distribution along the average value. This broadening may reflect the difference between halo nuclei and weakly bound nuclei. Similar to Fig. 1(e), the improved UFF is applied for the weakly

TABLE II. Value of the three parameters, I, M, and P, for tightly and weakly bound projectile systems. For the halo projectiles in $10 < V_B < 20$ MeV, only P was searched because of the small number of data points.

Barrier, V_B (MeV)	I	Tightly bound		Weakly bound		Halo	
		M	P	M	P	M	P
$V_B < 10$	0.69	1	3.66	1.55	0	1.55	0
$10 < V_B < 20$	0.89	1	2.38	1.15	0.28	1.15	0.71
$V_B > 20$	1.03	1	0.62	1.16	0.39	1.13	0.75

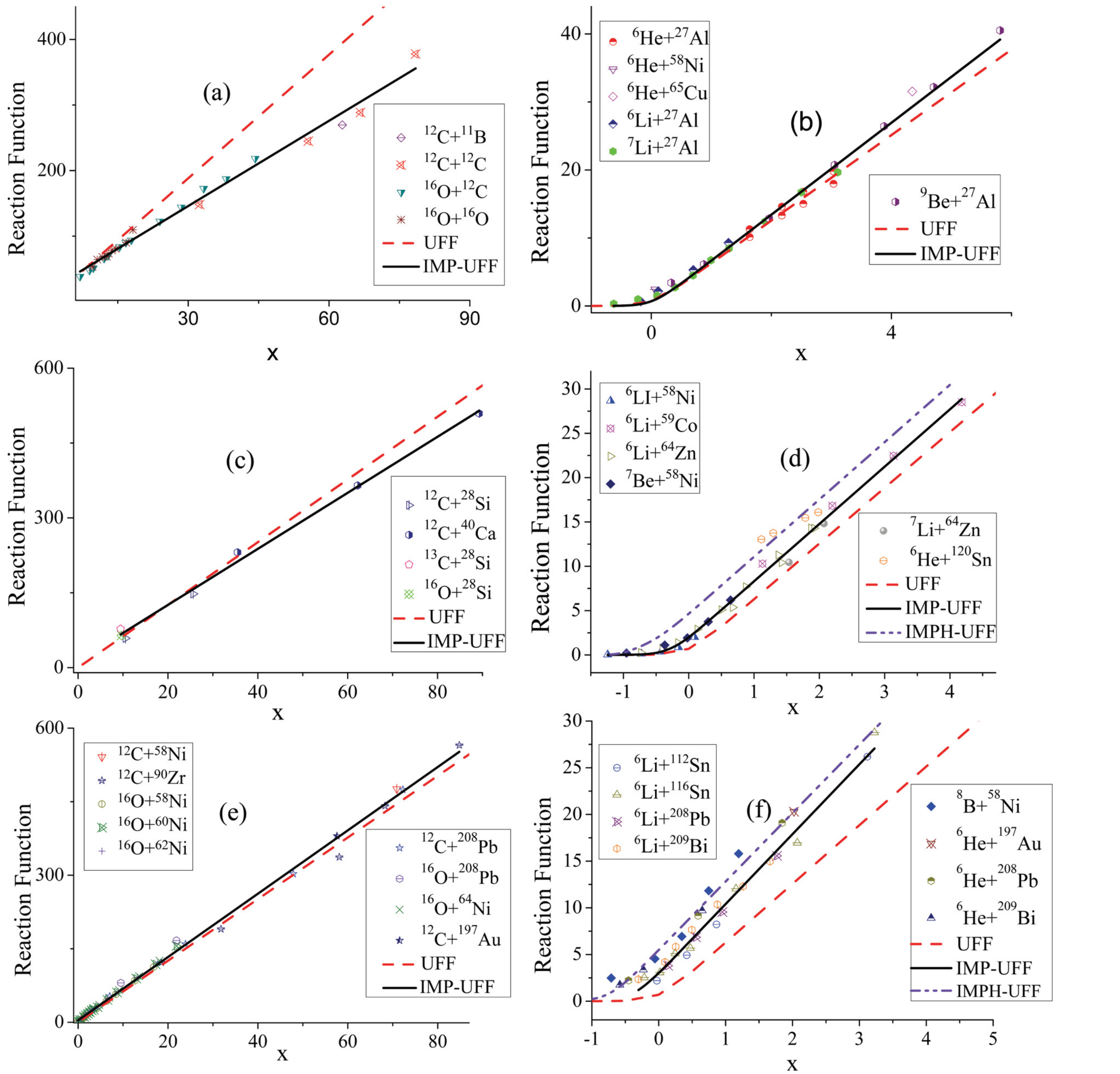

FIG. 1. (Color online) Reaction functions in collisions of tightly and weakly bound projectile systems. For comparison, we show the UFF and improved UFF (IMP-UFF). IMPH-UFF indicates the fitting of halo projectiles.

bound and halo projectiles, separately. The results are shown in Table II.

As described above, the reduced reaction function of the tightly bound, weakly bound, and halo projectile systems can be reproduced well by the improved UFF (IMP-UFF), which derives from Wong's formula. Therefore, Wong's formula can be reformulated by IMP-UFF. Using Eq. (4) in Eq. (1) we can get

$$\sigma_{\text{tot}} \simeq IMR_B^2 \frac{\hbar\omega}{2E} \ln\left[1 + \exp\left(\frac{2\pi(E - V_B + P')}{\hbar\omega}\right)\right], \quad (5)$$

where $P' = P\hbar\omega$ and V_B is the Coulomb barrier height. The value of these three parameters (I, M, and P) can be derived by fitting of the experimental data. Here, the parameter I gives the Coulomb barrier dependency of the modifications in the cross section; the parameter M indicates the enhancement of the cross section for weakly bound systems. For the tightly bound system, M was taken to be 1, assuming no couplings or a very weak coupling to the breakup channel; the parameter P modifies the Coulomb barrier height to adjust the collision energy where the reaction function starts to increase sharply. In Fig. 1 all experimental points of reaction cross sections are well reproduced by IMP-UFF. For different regions of Coulomb barrier energy, the extracted parameters show distinct trends.

The product of parameter I and M (IM) is uniquely determined by the reaction system for a given value of R_B. The R_B values used in this work are from SPP [9] and typical values are given in Table I, although their values depend slightly on the incident energy. The value of parameter I increases for the larger Coulomb barrier regions, indicating that the modification of the total reaction cross section increases when the target nucleus becomes heavier. The M value of a weakly bound system is larger than that of a tightly bound system. Thus, it can be inferred that the reaction mechanisms of tightly bound and weakly bound projectiles are different. Because of the low binding energy of the weakly bound projectile, it has larger breakup cross sections than those of a tightly bound projectile. This may be a reason that $M > 1$ for the weakly bound systems. It is interesting to note that the parameters M and P have large differences when one compares the tightly bound and the weakly bound systems. In other words, the modified formula with the parameters M and P can clearly distinguish between the tightly bound and the weakly bound systems.

A modified phenomenological formula of the total reaction cross section for low-energy systems is proposed. In this Brief Report, 71 sets of experimental data of tightly bound projectiles and 57 sets of weakly bound projectiles at low energies are compared. The experimental data of total reaction cross sections are reduced into the dimensionless total reaction functions. General trends in each Coulomb barrier region reflect different reaction mechanisms between the tightly bound and weakly bound projectile systems. The trends derived from the experimental data result in the reformulation of Wong's formula by adding three more parameters. We reached the following conclusions: (i) in the same energy region with the same projectile, as the target becomes heavier, the modified barrier radius becomes larger, hence the total reaction cross section becomes larger; and (ii) weakly bound projectiles have larger reaction cross sections because of the dynamic effects of weakly bound projectile systems, compared to that of tightly bound projectile systems.

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