

Inclusive breakup reactions with nonspectator fragments: Generalization of the Ichimura-Austern-Vincent sum rules

Jin Lei *

*School of Physics Science and Engineering, Tongji University, Shanghai 200092, China
and Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics,
Chinese Academy of Sciences, Huizhou 516000, Guangdong Province, China*



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The Ichimura-Austern-Vincent (IAV) sum-rule formalism for inclusive breakup reactions $a + A \rightarrow b + \text{anything}$ treats the detected fragment b as a spectator by replacing its interaction with the target by an optical potential. This assumption becomes questionable when b is a loosely bound composite particle such as a deuteron. I derive a generalization that removes the spectator approximation and retains b 's internal degrees of freedom, providing state-resolved inclusive cross sections. Within the distorted-wave Born approximation, all nonspectator effects enter through the source function via the operator $V_{bA} - U_{bA}$. The exact sum rule involves the full $x + A$ resolvent $(E_{x,0}^+ - H_{xA})^{-1}$, while a single-channel IAV-like expression is recovered only when the explicit target dependence of V_{bA} is neglected; post-prior equivalence is preserved in both cases, since V_{bA} cancels in the difference between the post- and prior-form residual interactions, and is verified explicitly in the single-channel limit. A key conceptual finding is that the standard IAV result for structureless b corresponds, under closure and a reduction of b 's internal space, to the *total* inclusive cross section summed over all of b 's internal states, rather than the cross section for b in a specific state. An operator-level estimate for $b = d$ on ^{208}Pb shows that the nonspectator correction is not a small perturbation at the nuclear surface. The present work is purely formal: it establishes the theoretical framework and identifies the relevant operators, while quantitative assessment of the cross-section impact awaits a full numerical evaluation of the source integrals.

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I. INTRODUCTION

Inclusive breakup reactions, in which a composite projectile a impinges on a target A and only one fragment b is detected in the exit channel while everything else is summed over, play an important role in nuclear reaction theory [1]. The reaction $a + A \rightarrow b + \text{anything}$, where $a = b + x$, includes a range of physical processes: elastic breakup (EBU), in which both fragments emerge with the target in its ground state; nonelastic breakup (NEB), also called breakup fusion, in which fragment x is absorbed by the target; and more complex rearrangement channels [2–4]. Because only fragment b is detected, the experiment integrates over all of these final-state configurations, making a fully exclusive theoretical description unnecessary and motivating sum-rule approaches that exploit this inclusiveness.

The theoretical framework for inclusive breakup was developed in a series of papers by Udagawa, Tamura, and Li [5,6] (the prior-form or UT approach), and independently by Austern and Vincent [7] and Ichimura, Austern, and Vincent [8] (the post-form or AV approach). These two lines of work, now collectively referred to as the IAV formalism, derive closed-form sum rules for the inclusive breakup cross section by performing the sum over unobserved final states

analytically through quantum mechanical completeness. The central result of the post-form derivation is that the inclusive nonelastic breakup cross section can be expressed in terms of the imaginary part of an optical-model Green's function G_x for the unobserved fragment x in the target field, acting on a source function that encodes the breakup dynamics. Ichimura, Austern, and Vincent demonstrated that the post and prior forms of the sum rule are equivalent, provided certain surface terms vanish upon appropriate regularization [8,9].

A key assumption underlying the IAV formalism is that the detected fragment b acts as a *spectator*. In the model Hamiltonian of Ref. [8] [Eq. (2.1) therein], the interaction V_{bA} between b and the target is replaced by an optical potential U_{bA} that depends only on the center-of-mass coordinate of b and does not excite the target. The distorted wave $\chi_b^{(-)}$ for b in the exit channel is generated by a (generally different) optical potential U_{bB} . The post-form residual interaction in the IAV model is then $V_{\text{post}}^{\text{IAV}} = V_{bx} + U_{bA} - U_{bB}$, which does not depend on the target internal coordinates since neither U_{bA} nor U_{bB} excites the target. This property is essential for the Feshbach optical reduction [10,11] that produces the closed-form sum rule involving G_x . In addition, b is treated as structureless, eliminating any role for its internal degrees of freedom. For reactions where b is indeed a tightly bound, compact fragment (such as an α particle), the spectator approximation is well justified, and the IAV formalism has been applied with considerable success [12–20].

*Contact author: jinl@tongji.edu.cn

There exist, however, physically important reactions in which the detected fragment b is itself a loosely bound composite particle [21,22]. A prominent example is inclusive deuteron production in reactions induced by ${}^6\text{Li}$, ${}^7\text{Li}$, or triton projectiles, where $b = d$ (deuteron) and x is a heavier fragment absorbed by the target. The deuteron, with a binding energy of only 2.224 MeV and a root-mean-square proton-neutron separation of approximately 3.9 fm, is susceptible to polarization, excitation, and breakup in the nuclear and Coulomb fields of the target [23–25]. An even more extreme case arises in ${}^9\text{Be}$ -induced reactions [26], where the detected fragment $b = {}^8\text{Be}$ ($\alpha + \alpha$) is itself unbound and necessarily breaks up after neutron transfer to the target. In such cases, the replacement $V_{bA} \rightarrow U_{bA}$ is a significant approximation, because the full interaction V_{bA} depends on the internal coordinates of b and can couple b 's internal excitations to target excitations. Ichimura, Austern, and Vincent themselves noted this limitation, remarking in Sec. V of Ref. [8] that “the importance of excitation during breakup should be no surprise” and that “a reduced theory of a many-body system, in terms of the coordinates of only a few ‘active’ particles, requires the careful introduction of effective interactions.” An effective three-body Hamiltonian analysis based on double Feshbach projection [27] shows that the standard additive model for composite projectiles misses nonadditive induced interactions whose omission can significantly distort reaction cross sections. Despite this recognition, no systematic generalization of the IAV sum rules to the nonspectator case has been developed.

Potel *et al.* [12,13] implemented the prior-form sum rule and demonstrated that it provides a convergent, practical scheme for computing inclusive breakup cross sections. Lei and Moro [14–16] extended these calculations to many reactions and showed good agreement with experimental data for (d, p) -type and $({}^6\text{Li}, \alpha)$ -type reactions, and further demonstrated that the IAV framework can explain complete fusion suppression and partial fusion in weakly bound systems [18,19]. These implementations exclusively use the prior form, because the post-form matrix element suffers from a convergence issue related to the disconnected part of the elastic breakup amplitude [8,28]. Carlson, Frederico, and Hussein [29] extended the spectator model to three-fragment projectiles within a four-body framework, obtaining inclusive elastic and nonelastic breakup cross sections for weakly bound systems such as Borromean nuclei. More recently, Neoh *et al.* [30] formulated inclusive breakup within the eikonal reaction theory, and Deltuva [31] developed a Faddeev-based inclusive breakup framework that treats all three-body channels on equal footing.

In this paper, I derive a generalization of the IAV inclusive breakup formalism that removes the spectator approximation for fragment b . A key conceptual point, developed in Sec. IV, is that the standard IAV treatment with structureless b admits a natural correspondence, under closure and structureless-reduction approximations, with the *total* inclusive cross section summed over all of b 's internal states, rather than the state-resolved cross section for detecting b in a specific state. The present formalism introduces the projection onto b 's ground state explicitly, providing access

to state-resolved observables. At the exact distorted-wave Born approximation (DWBA) level the resulting sum rule involves the full $x + A$ resolvent, whereas the familiar optical-propagator form reappears only in a reduced single-channel limit. Following the logical structure of the original IAV paper, I first develop the post-form sum rule in Sec. II, then the prior-form sum rule and post-prior equivalence in Sec. III. In both forms, the nonspectator correction is controlled by the single operator $V_{bA} - U_{bA}$. In Sec. V, I present a tidal estimate for the deuteron case, quantifying the differential pull of the target field across the deuteron's internal extent, which is the physical content of $V_{bA} - U_{bA}$. I discuss practical implications in Sec. VI and summarize in Sec. VII.

II. POST-FORM SUM RULE

I consider the inclusive breakup reaction $a + A \rightarrow b + \text{anything}$, where the projectile a is a bound state of fragments b and x , so that $a = b + x$. Fragment b is composite with internal degrees of freedom ζ , described by the internal Hamiltonian $h_b(\zeta)$ with eigenstates $h_b|\phi_n\rangle = \varepsilon_n|\phi_n\rangle$. The ground state $|\phi_0\rangle$ with energy ε_0 is the state in which b is detected experimentally, while the states with $n \geq 1$ include excited bound states and continuum states corresponding to the breakup of b . These eigenstates form a complete set, $\sum_n |\phi_n\rangle\langle\phi_n| = \mathbf{1}_\zeta$, in b 's internal Hilbert space.

The full Hamiltonian for the $a + A$ system is

$$H = H_A(\xi) + h_b(\zeta) + K_b + K_x + V_{bx} + V_{bA} + V_{xA}, \quad (1)$$

where $H_A(\xi)$ is the internal Hamiltonian of the target nucleus with ground state $H_A\Phi_A = E_A\Phi_A$, ξ denotes the target internal coordinates, K_b and K_x are the kinetic energies defined below, and $V_{bx}(\mathbf{r}_{bx}, \zeta)$, $V_{bA}(\mathbf{r}_{bA}, \zeta, \xi)$, and $V_{xA}(\mathbf{r}_x, \xi)$ are the pairwise interactions. The Hamiltonian is written in the center-of-mass frame using relative coordinates, retaining the full target mass with no heavy-target approximation. I use the position $\mathbf{r}_x$ of x relative to the target A , the position $\mathbf{r}_{bA}$ of b relative to A , and the position $\mathbf{r}_{bB}$ of b relative to the center of mass of the residual $B = x + A$. The b - x relative coordinate is $\mathbf{r}_{bx} = \mathbf{r}_{bA} - \mathbf{r}_x$, and the projectile center of mass relative to A is $\mathbf{r}_a = (m_b\mathbf{r}_{bA} + m_x\mathbf{r}_x)/m_a$, with $m_a = m_b + m_x$. The two b coordinates differ by the recoil of the residual, $\mathbf{r}_{bA} = \mathbf{r}_{bB} + \frac{m_x}{m_x + M_A}\mathbf{r}_x$, and coincide as $M_A \rightarrow \infty$. Each binary interaction is local in its respective separation: V_{bA} and U_{bA} in $\mathbf{r}_{bA}$ (with V_{bA} additionally parametric in ζ and ξ , as detailed below), V_{xA} in $\mathbf{r}_x$, V_{bx} in $\mathbf{r}_{bx}$ (and in ζ), and the auxiliary distorting potential U_{bB} (defined below) in $\mathbf{r}_{bB}$. The kinetic energies form two equivalent Jacobi pairs: in the exit channel, K_x for x relative to A and K_b for b relative to B , conjugate to $\mathbf{r}_x$ and $\mathbf{r}_{bB}$; in the entrance channel, K_{bx} for the b - x motion and K_a for the projectile $a = b + x$ relative to A , conjugate to $\mathbf{r}_{bx}$ and $\mathbf{r}_a$. Each pair diagonalizes the kinetic energy with no cross term, and the two are connected by the exact Jacobi transformation $K_b + K_x = K_{bx} + K_a$ used in the prior form (Sec. III). Following the standard IAV notation, interactions carry the pair of interacting bodies as subscripts, as in U_{bA} and U_{bB} , whereas kinetic energies and distorted waves carry only the fragment label, the partner being fixed by the channel; thus K_b and $\chi_b^{(-)}$ both refer to the motion of b relative to B in the

exit channel. To keep the expressions compact, the arguments of operators and wave functions are displayed only at their first appearance or where the explicit dependence is essential to the argument, and are suppressed otherwise. The interaction V_{bx} depends on $\mathbf{r}_{bx}$ and on ζ because the binding between b and x is sensitive to b 's internal configuration. The interaction V_{bA} depends on b 's center-of-mass position $\mathbf{r}_{bA}$, on b 's internal coordinates ζ (because the individual constituents of b interact with the target at different positions), and on ξ (because it can excite the target). The projectile bound-state wave function $\varphi_a(\mathbf{r}_{bx}, \zeta)$ satisfies

$$(h_b + K_{bx} + V_{bx})\varphi_a = -\epsilon_a\varphi_a, \quad (2)$$

where K_{bx} is the b - x relative kinetic energy and $\epsilon_a > 0$ is the projectile binding energy.

For comparison, the IAV model Hamiltonian [8] is $H^{\text{IAV}} = H_A + K_b + K_x + V_{xA} + U_{bA} + V_{bx}$, which differs from Eq. (1) in that V_{bA} has been replaced by an optical potential U_{bA} and h_b is absent.

To construct the post-form DWBA, I introduce the exit-channel Hamiltonian

$$H_{\text{exit}} = h_b + K_b + U_{bB} + H_{xA}, \quad (3)$$

where $H_{xA} = H_A + K_x + V_{xA}$ is the full $x + A$ Hamiltonian and U_{bB} is an auxiliary optical potential that generates the distorted wave $\chi_b^{(-)}$ for fragment b . The exit-channel Hamiltonian is separable in b 's degrees of freedom ($h_b + K_b + U_{bB}$) and the $x + A$ system (H_{xA}), so the exit-channel eigenstates factorize as $|\chi_b^{(-)}(\mathbf{k}_b)\rangle |\phi_n(\zeta)\rangle |\Psi_{xA}^c\rangle$, where $H_{xA}|\Psi_{xA}^c\rangle = E^c|\Psi_{xA}^c\rangle$. The post-form residual interaction is

$$V_{\text{post}} = H - H_{\text{exit}} = V_{bx} + V_{bA} - U_{bB}. \quad (4)$$

In the IAV model, where V_{bA} is replaced by U_{bA} , the post-form residual is $V_{\text{post}}^{\text{IAV}} = V_{bx} + U_{bA} - U_{bB}$, which does not depend on the target coordinates ξ since neither U_{bA} nor U_{bB} excites the target. In the present case, V_{post} contains the full interaction V_{bA} , which depends on $\mathbf{r}_{bA}$, ζ , and ξ .

The DWBA transition amplitude for detecting b in its ground state ϕ_0 while the $x + A$ system is in eigenstate c is

$$T_{0,c} = \langle \chi_b^{(-)} \phi_0 | \Psi_{xA}^c | V_{\text{post}} | \chi_a^{(+)} \varphi_a \Phi_A \rangle, \quad (5)$$

where $\chi_a^{(+)}$ is the entrance-channel distorted wave. The doubly differential inclusive cross section, summed over all unobserved final states of the $x + A$ system, is

$$\left. \frac{d^2\sigma}{dE_b d\Omega_b} \right|_{\text{post}} = \frac{(2\pi)^4}{v_a} \sum_c |T_{0,c}|^2 \delta(E_{x,0} - E^c), \quad (6)$$

where v_a is the a - A relative velocity, E^c is the eigenenergy of the $x + A$ final state Ψ_{xA}^c , and $E_{x,0} = E - E_b - \epsilon_0$ is the energy available to the $x + A$ system when b is in its ground state; here E is the total energy of the system in the center-of-mass frame and $E_b = \hbar^2 k_b^2 / (2\mu_b)$ is the kinetic energy of b , with μ_b the b -($x+A$) reduced mass.

I define the source function by integrating out b 's degrees of freedom,

$$|\rho_0\rangle = \langle \phi_0(\zeta) | \chi_b^{(-)}(\mathbf{r}_{bB}) | V_{\text{post}} | \chi_a^{(+)} \varphi_a \Phi_A \rangle, \quad (7)$$

which is obtained by projecting $V_{\text{post}}|\chi_a^{(+)}\varphi_a\Phi_A\rangle$ onto b 's ground state and the distorted wave $\chi_b^{(-)}$, integrating over b 's spatial coordinate $\mathbf{r}_{bB}$ and internal coordinate ζ . Here and below, a bra of the form $\langle \phi_0 | \chi_b^{(-)} |$ denotes this partial projection over b 's center-of-mass and internal coordinates, leaving a ket in the $x + A$ Hilbert space. The result $|\rho_0\rangle$ is a state in the $x + A$ Hilbert space depending on $\mathbf{r}_x$ and ξ . In terms of ρ_0 , the T -matrix becomes $T_{0,c} = \langle \Psi_{xA}^c | \rho_0 \rangle$, and the spectral identity $\sum_c |\Psi_{xA}^c\rangle \langle \Psi_{xA}^c | \delta(E_{x,0} - E^c) = -\pi^{-1} \text{Im} (E_{x,0}^+ - H_{xA})^{-1}$, where the superscript $+$ denotes the limit $E_{x,0} + i0^+$ (outgoing-wave boundary conditions, here and throughout), yields

$$\left. \frac{d^2\sigma}{dE_b d\Omega_b} \right|_{\text{post}} = -\frac{(2\pi)^4}{\pi v_a} \text{Im} \langle \rho_0 | (E_{x,0}^+ - H_{xA})^{-1} | \rho_0 \rangle. \quad (8)$$

This is exact within the DWBA. The propagator $(E_{x,0}^+ - H_{xA})^{-1}$ is the standard $x + A$ resolvent that also underlies the original IAV derivation, where its ground-state optical reduction yields the Green's function G_x . All nonspectator physics enters through the source ρ_0 .

Since $V_{\text{post}} = (V_{bx} + U_{bA} - U_{bB}) + (V_{bA} - U_{bA})$, the source decomposes as $|\rho_0\rangle = |\rho_0^{(\text{sp})}\rangle + |\rho_0^{(\text{ns})}\rangle$, where the spectator part

$$|\rho_0^{(\text{sp})}\rangle = \langle \phi_0 | \chi_b^{(-)} | (V_{bx} + U_{bA} - U_{bB}) | \chi_a^{(+)} \varphi_a \Phi_A \rangle \quad (9)$$

is the spectator source projected onto b 's ground state (the $\mathcal{P}$ -channel component of the IAV-type source), and the non-spectator part

$$|\rho_0^{(\text{ns})}\rangle = \langle \phi_0 | \chi_b^{(-)} | (V_{bA} - U_{bA}) | \chi_a^{(+)} \varphi_a \Phi_A \rangle \quad (10)$$

involves the correction $V_{bA} - U_{bA}$, the difference between the true b - A interaction and the optical model used in the IAV framework. Physically, this source receives contributions because the projectile wave function $\varphi_a(\mathbf{r}_{bx}, \zeta)$ is a superposition of b 's internal eigenstates, $\varphi_a = \sum_n a_n(\mathbf{r}_{bx}) \phi_n(\zeta)$. The operator $V_{bA} - U_{bA}$ connects these components to the detected ground state ϕ_0 through its matrix elements $\langle \phi_0 | V_{bA} | \phi_n \rangle$: the diagonal element ($n = 0$) corrects for the difference between the true ground-state interaction and the optical potential, while the off-diagonal elements ($n \geq 1$) transfer amplitude from excited components of b within the projectile back to the ground state during the interaction with the target. A detailed decomposition is given in Sec. IV.

The spectator source has a property essential for the optical reduction. Because none of V_{bx} , U_{bA} , or U_{bB} depends on ξ , and the only ξ dependence in the integrand comes from $\Phi_A(\xi)$, the result factorizes as $\rho_0^{(\text{sp})}(\mathbf{r}_x, \xi) = \tilde{\rho}_0(\mathbf{r}_x) \Phi_A(\xi)$, where the reduced source

$$\tilde{\rho}_0(\mathbf{r}_x) = \int d\mathbf{r}_{bB} d\zeta \chi_b^{(-)*} \phi_0^* (V_{bx} + U_{bA} - U_{bB}) \chi_a^{(+)} \varphi_a \quad (11)$$

depends on $\mathbf{r}_x$ alone; here U_{bA} is evaluated at $\mathbf{r}_{bA} = \mathbf{r}_{bB} + \frac{m_x}{m_x + M_A} \mathbf{r}_x$ via the recoil relation while the integration runs over $\mathbf{r}_{bB}$, and the entrance-channel functions carry their own $\mathbf{r}_x$ dependence through $\mathbf{r}_a = (m_b \mathbf{r}_{bA} + m_x \mathbf{r}_x)/m_a$ in $\chi_a^{(+)}$ and $\mathbf{r}_{bx} = \mathbf{r}_{bA} - \mathbf{r}_x$ in φ_a . This factorization enables the Feshbach optical reduction $\langle \Phi_A | (E_{x,0}^+ - H_{xA})^{-1} | \Phi_A \rangle = G_x = (E_{x,0}^+ - E_A -$

$K_x - U_x)^{-1}$, where U_x is the Feshbach optical potential for x in the target field.

The nonspectator source does not factorize in this way. In coordinate representation,

$$\begin{aligned} \rho_0^{(\text{nsp})}(\mathbf{r}_x, \xi) &= \int d\mathbf{r}_{bB} d\zeta \chi_b^{(-)*} \phi_0^* \\ &\times [V_{bA}(\mathbf{r}_{bA}, \zeta, \xi) - U_{bA}(\mathbf{r}_{bA})] \chi_a^{(+)} \varphi_a \Phi_A(\xi). \end{aligned} \quad (12)$$

The product $V_{bA} \Phi_A(\xi)$ generates components on excited target states, so $\rho_0^{(\text{nsp})}$ is entangled in $\mathbf{r}_x$ and ξ and the optical reduction does not apply.

Substituting into Eq. (8), the cross section separates into three terms:

$$\left. \frac{d^2\sigma}{dE_b d\Omega_b} \right|_{\text{post}} = \sigma_{\text{sp}} + \sigma_{\text{cross}} + \sigma_{\text{nsp}}. \quad (13)$$

Here and below, each σ with sub- or superscripts denotes a component of the same doubly differential cross section $d^2\sigma/(dE_b d\Omega_b)$, with the differential arguments suppressed. The spectator term is

$$\sigma_{\text{sp}} = -\frac{(2\pi)^4}{\pi v_a} \text{Im} \langle \tilde{\rho}_0 | G_x | \tilde{\rho}_0 \rangle, \quad (14)$$

which has the same structure as Eq. (2.11) of Ref. [8]. The cross and nonspectator terms involve the full $x + A$ resolvent $G_{\text{full}} = (E_{x,0}^+ - H_{xA})^{-1}$ rather than the optical Green's function G_x , because $\rho_0^{(\text{nsp})}$ has nontrivial ξ dependence:

$$\sigma_{\text{cross}} = -\frac{(2\pi)^4}{\pi v_a} \text{Im} [(\rho_0^{(\text{sp})} | G_{\text{full}} | \rho_0^{(\text{nsp})}) + (\text{sp} \leftrightarrow \text{nsp})], \quad (15)$$

$$\sigma_{\text{nsp}} = -\frac{(2\pi)^4}{\pi v_a} \text{Im} \langle \rho_0^{(\text{nsp})} | G_{\text{full}} | \rho_0^{(\text{nsp})} \rangle. \quad (16)$$

To evaluate these, one may expand $\rho_0^{(\text{nsp})}$ in target eigenstates as $|\rho_0^{(\text{nsp})}\rangle = \sum_{\alpha} |\Phi_{\alpha}\rangle |\varrho_{\alpha}(\mathbf{r}_x)\rangle$, leading to $\text{Im} \sum_{\alpha, \beta} \langle \varrho_{\alpha} | [\mathbf{G}_x]_{\alpha\beta} | \varrho_{\beta} \rangle$, where $[\mathbf{G}_x]_{\alpha\beta} = \langle \Phi_{\alpha} | G_{\text{full}} | \Phi_{\beta} \rangle$ is the coupled-channel Green's function. Equations (14)–(16) constitute the DWBA-exact result derived from the full Hamiltonian; the cross and nonspectator terms require G_{full} and, in general, coupled target channels, whereas an IAV-like single-channel expression reappears only after an additional simplifying assumption is made (see below).

The spectator term deserves a further remark regarding its relationship to the standard IAV formula. The present source $\tilde{\rho}_0$ [Eq. (11)] involves the projection $\langle \phi_0(\zeta) |$ and the composite projectile wave function $\varphi_a(\mathbf{r}_{bx}, \zeta)$, whereas the standard IAV source uses a structureless wave function $\varphi_a^{\text{sl}}(\mathbf{r}_{bx})$ with no ζ variable, obtained by collapsing b 's internal Hilbert space to a single degree of freedom. The passage from $\tilde{\rho}_0$ to the corresponding structureless source $\tilde{\rho}_0^{\text{sl}}(\mathbf{r}_x) = \int d\mathbf{r}_{bB} \chi_b^{(-)*} (V_{bx}^{\text{eff}} + U_{bA} - U_{bB}) \chi_a^{(+)} \varphi_a^{\text{sl}}$ constitutes an additional *structureless-reduction* approximation, distinct from the spectator replacement $V_{bA} \rightarrow U_{bA}$. The full approximation chain from the exact Hamiltonian to the standard IAV thus involves two steps: (i) replacing V_{bA} by U_{bA} (spectator approximation), and (ii) collapsing b 's internal space by identifying $\langle \phi_0 | \varphi_a \rangle_{\zeta} \rightarrow \varphi_a^{\text{sl}}$ and $\langle \phi_0 | V_{bx} | \varphi_a \rangle_{\zeta} \rightarrow V_{bx}^{\text{eff}} \varphi_a^{\text{sl}}$ (structureless reduction). The present formalism retains both h_b and the full

V_{bA} ; Eq. (14) matches the standard IAV form only after the structureless reduction is also applied.

Following IAV, the spectator term is further decomposed into EBU and NEB contributions using the operator identity [8]

$$\text{Im} G_x = (1 + G_x^{\dagger} U_x^{\dagger}) \text{Im} G_0 (1 + U_x G_x) + G_x^{\dagger} W_x G_x, \quad (17)$$

where $G_0 = (E_{x,0}^+ - E_A - K_x)^{-1}$ is the free Green's function and $W_x = \text{Im} U_x$ is the absorptive part of the optical potential. When inserted into Eq. (14), the first term on the right-hand side gives the spectator elastic-breakup component, in which x scatters elastically off A while remaining in the continuum. The second term gives the spectator nonelastic-breakup (breakup fusion) component, in which x is absorbed by the target. Inserting Eq. (17) into Eq. (14) yields the EBU-NEB decomposition of the spectator term,

$$\sigma_{\text{sp}} = \sigma_{\text{sp}}^{\text{EBU}} + \sigma_{\text{sp}}^{\text{NEB}}. \quad (18)$$

The NEB part is

$$\sigma_{\text{sp}}^{\text{NEB}} = -\frac{(2\pi)^4}{\pi v_a} \langle \tilde{\rho}_0 | G_x^{\dagger} W_x G_x | \tilde{\rho}_0 \rangle. \quad (19)$$

This has the same operator structure as the nonelastic breakup formula computed by existing inclusive breakup codes [12,14] (Eq. (2.21) of Ref. [8]); the two coincide after the structureless reduction discussed above. The EBU part $\sigma_{\text{sp}}^{\text{EBU}}$ is the spectator EBU component generated by the first term of Eq. (17).

The spectator or nonspectator classification and the EBU or NEB classification are logically distinct: the former concerns how the source is generated; the latter concerns which $x + A$ final-state channel that source feeds. Once the source is modified by $V_{bA} - U_{bA}$, both the EBU and NEB projections of the observed intact- b cross section can change. At the level of the full DWBA result, both elastic and nonelastic breakup receive spectator, cross, and nonspectator contributions:

$$\sigma^{\text{EBU}} = \sigma_{\text{sp}}^{\text{EBU}} + \sigma_{\text{cross}}^{\text{EBU}} + \sigma_{\text{nsp}}^{\text{EBU}} \quad (20)$$

and

$$\sigma^{\text{NEB}} = \sigma_{\text{sp}}^{\text{NEB}} + \sigma_{\text{cross}}^{\text{NEB}} + \sigma_{\text{nsp}}^{\text{NEB}}. \quad (21)$$

For elastic breakup, σ^{EBU} corresponds to the exclusive final channel $b(\phi_0) + x + A(\Phi_A)$, which can be fed by both the spectator and nonspectator sources together with their interference. For nonelastic breakup, σ^{NEB} likewise receives all three contributions; because NEB is the quantity that the IAV formalism is designed to provide, the nonspectator correction is most consequential here. An EBU-NEB decomposition analogous to Eq. (17) can be performed for each matrix element of the coupled-channel Green's function $[\mathbf{G}_x]_{\alpha\beta}$, but it involves inelastic target channels and does not reduce to the single-channel form of Eq. (19). In practice, the spectator elastic-breakup contribution $\sigma_{\text{sp}}^{\text{EBU}}$ is typically computed separately via the continuum-discretized coupled-channels (CDCC) method [25,32–34] or direct DWBA, independent of the IAV sum rule.

The additional simplification is to construct V_{bA} from nucleon optical potentials [35] and neglect any explicit ξ

dependence. Then the nonspectator source is again proportional to $\Phi_A(\xi)$, just like the spectator source, so the same Feshbach optical reduction applies to all three terms. Defining the nonspectator reduced source as

$$\tilde{\rho}_0^{(\text{nsp})}(\mathbf{r}_x) = \int d\mathbf{r}_{bB} d\zeta \chi_b^{(-)*} \phi_0^* (V_{bA} - U_{bA}) \chi_a^{(+)} \varphi_a, \quad (22)$$

the total inclusive cross section in this reduced single-channel limit takes the form

$$\left. \frac{d^2\sigma}{dE_b d\Omega_b} \right|_{\text{post}} = -\frac{(2\pi)^4}{\pi v_a} \text{Im} \langle \tilde{\rho}_0^{\text{tot}} | G_x | \tilde{\rho}_0^{\text{tot}} \rangle, \quad (23)$$

where $\tilde{\rho}_0^{\text{tot}} = \tilde{\rho}_0 + \tilde{\rho}_0^{(\text{nsp})}$. In this reduced single-channel limit, the EBU-NEB decomposition of Eq. (17) applies to the full cross section, and the NEB part becomes

$$\sigma^{\text{NEB}} = -\frac{(2\pi)^4}{\pi v_a} \langle \tilde{\rho}_0^{\text{tot}} | G_x^\dagger W_x G_x | \tilde{\rho}_0^{\text{tot}} \rangle. \quad (24)$$

This is the central *practical* result of this work; the DWBA-exact result remains Eqs. (14)–(16). It states that, once the target dependence of V_{bA} has been suppressed, the nonelastic breakup cross section has the same operator structure as Eq. (19), with the source $\tilde{\rho}_0$ replaced by the total source $\tilde{\rho}_0^{\text{tot}}$. After the additional structureless reduction discussed above, this becomes the standard IAV expression with a modified source. In existing IAV codes that compute $\langle \tilde{\rho}_0 | G_x^\dagger W_x G_x | \tilde{\rho}_0 \rangle$, the propagator and NEB projection remain unchanged; the extension consists of replacing $\tilde{\rho}_0$ by $\tilde{\rho}_0^{\text{tot}}$. Constructing the new source $\tilde{\rho}_0^{(\text{nsp})}$ is itself a nontrivial step, discussed in Sec. VI.

III. PRIOR-FORM SUM RULE

While the post-form derivation provides the most transparent view of how the spectator approximation enters and how it is removed, practical implementations of the IAV formalism use the prior form [12,14] because the post-form matrix element suffers from a convergence problem [8,28]. The entrance-channel Hamiltonian is $H_{\text{ent}} = H_A + K_a + U_a + h_b + K_{bx} + V_{bx}$, where $K_b + K_x = K_a + K_{bx}$ by Jacobi transformation and U_a is the entrance-channel optical potential. The prior-form residual interaction is

$$V_{\text{prior}} = H - H_{\text{ent}} = V_{xA} + V_{bA} - U_a. \quad (25)$$

In the standard IAV with $V_{bA} \rightarrow U_{bA}$, this reduces to $V_{\text{prior}}^{\text{IAV}} = V_{xA} + U_{bA} - U_a$.

The prior-form DWBA T -matrix $T'_{0,c} = \langle \chi_b^{(-)} \phi_0 \Psi_{xA} | V_{\text{prior}} | \chi_a^{(+)} \varphi_a \Phi_A \rangle$ defines a prior source $|\rho_0'\rangle = \langle \phi_0 \chi_b^{(-)} | V_{\text{prior}} | \chi_a^{(+)} \varphi_a \Phi_A \rangle$ whose contribution to the NEB cross section is the UT term [5,36]. Because the exit-channel optical potential U_{bB} is complex, the prior-form DWBA does not coincide with the post-form result; the difference is made up by the nonorthogonality (NO) and interference (IN) corrections identified in Ref. [36]. In the standard single-channel IAV setting, the full IAV-equivalent prior-form NEB is $\sigma^{\text{NEB}} = \sigma_{\text{UT}}^{\text{NEB}} + \sigma_{\text{NO}}^{\text{NEB}} + \sigma_{\text{IN}}^{\text{NEB}}$, and the nonspectator correction modifies both the UT and IN terms, as detailed below.

I now restrict to the same reduced single-channel limit used in Sec. II, in which the target dependence of V_{bA} is suppressed and the projection onto the target ground state introduces the optical propagator $G_x = (E_{x,0}^+ - E_A - K_x - U_x)^{-1}$. In this limit, the reduced spectator UT source is built from the operator $U_x + U_{bA} - U_a$. Since none of these potentials depends on the target coordinates ξ , the spectator UT source factorizes in target space, just as in the post form. The reduced UT source is

$$\tilde{\rho}_0^{\text{UT}}(\mathbf{r}_x) = \int d\mathbf{r}_{bB} d\zeta \chi_b^{(-)*} \phi_0^* (U_x + U_{bA} - U_a) \chi_a^{(+)} \varphi_a, \quad (26)$$

and the spectator UT NEB cross section is [5,6,36]

$$\sigma_{\text{UT}}^{\text{NEB}} = -\frac{(2\pi)^4}{\pi v_a} \langle \tilde{\rho}_0^{\text{UT}} | G_x^\dagger W_x G_x | \tilde{\rho}_0^{\text{UT}} \rangle, \quad (27)$$

which has the same single-channel operator structure as the post-form NEB [Eq. (19)]. The standard IAV prior-form result is recovered after the structureless reduction $\langle \phi_0 | \varphi_a \rangle_\xi \rightarrow \varphi_a^{\text{sl}}$.

The nonspectator correction enters by replacing U_{bA} with V_{bA} in the UT source, giving the total reduced prior source

$$\tilde{\rho}_0^{\text{tot}}(\mathbf{r}_x) = \int d\mathbf{r}_{bB} d\zeta \chi_b^{(-)*} \phi_0^* (U_x + V_{bA} - U_a) \chi_a^{(+)} \varphi_a, \quad (28)$$

which decomposes as $\tilde{\rho}_0^{\text{tot}} = \tilde{\rho}_0^{\text{UT}} + \tilde{\rho}_0^{(\text{nsp})}$, where the nonspectator prior source

$$\tilde{\rho}_0^{(\text{nsp})}(\mathbf{r}_x) = \int d\mathbf{r}_{bB} d\zeta \chi_b^{(-)*} \phi_0^* (V_{bA} - U_{bA}) \chi_a^{(+)} \varphi_a \quad (29)$$

involves the same operator $V_{bA} - U_{bA}$ as in the post form [Eq. (10)]. The total prior-form UT NEB cross section, including the nonspectator correction, is therefore

$$\sigma_{\text{UT}}^{\text{NEB,tot}} = -\frac{(2\pi)^4}{\pi v_a} \langle \tilde{\rho}_0^{\text{tot}} | G_x^\dagger W_x G_x | \tilde{\rho}_0^{\text{tot}} \rangle, \quad (30)$$

which has the same structure as the post-form result [Eq. (24)] with the source $\tilde{\rho}_0^{\text{tot}}$ replaced by $\tilde{\rho}_0^{\text{tot}}$.

The NO and IN corrections that complete the prior-form NEB involve the NO overlap between entrance and exit channels, projected onto b 's ground state:

$$\psi_0^{\text{NO}}(\mathbf{r}_x) = \int d\mathbf{r}_{bB} d\zeta \chi_b^{(-)*}(\mathbf{r}_{bB}) \phi_0^*(\zeta) \chi_a^{(+)}(\mathbf{r}_a) \varphi_a(\mathbf{r}_{bx}, \zeta). \quad (31)$$

The NO term is the Hussein-McVoy (HM) contribution [37]. The NO and IN contributions to the NEB cross section are [8,36,37]

$$\sigma_{\text{NO}}^{\text{NEB}} = -\frac{(2\pi)^4}{\pi v_a} \langle \psi_0^{\text{NO}} | W_x | \psi_0^{\text{NO}} \rangle, \quad (32)$$

$$\sigma_{\text{IN}}^{\text{NEB}} = -\frac{(2\pi)^4}{\pi v_a} 2 \text{Re} \langle \psi_0^{\text{NO}} | W_x G_x | \tilde{\rho}_0^{\text{tot}} \rangle. \quad (33)$$

The NO term is pure spectator: it depends only on ψ_0^{NO} , which involves no interaction and therefore carries no V_{bA} dependence. The IN term, by contrast, contains the total prior source $\tilde{\rho}_0^{\text{tot}} = \tilde{\rho}_0^{\text{UT}} + \tilde{\rho}_0^{(\text{nsp})}$ and therefore decomposes into spectator and nonspectator contributions.

Post-prior equivalence. The difference of the post and prior residual interactions is $V_{\text{post}} - V_{\text{prior}} = V_{bx} - V_{xA} + U_a - U_{bB}$, where V_{bA} cancels exactly: whatever nonspectator effects V_{bA} introduces, it does so equally in both forms. The post-prior identity that connects the two representations is (cf. Eq. (4.18) of Ref. [8])

$$G_x |\tilde{\rho}_0^{\text{UT}}\rangle = G_x |\tilde{\rho}_0\rangle - |\psi_0^{\text{NO}}\rangle, \quad (34)$$

where $\tilde{\rho}_0$ is the post-form source [Eq. (11)]. Substituting this identity into the UT + NO + IN decomposition and collecting terms recovers the post-form NEB result [Eq. (19)], establishing that $\sigma_{\text{UT}}^{\text{NEB}} + \sigma_{\text{NO}}^{\text{NEB}} + \sigma_{\text{IN}}^{\text{NEB}} = \sigma_{\text{sp}}^{\text{NEB}}$ for the spectator sector. Identity (34) connects only the spectator UT and post sources. The nonspectator reduced source $\tilde{\rho}_0^{(\text{nsp})}$ [Eq. (29)] is built from the same operator $V_{bA} - U_{bA}$ and the same projection $\langle\phi_0|\chi_b^{(-)}|\cdots|\chi_a^{(+)}\varphi_a\rangle$ as the post-form nonspectator source $\tilde{\rho}_0^{(\text{nsp})}$ [Eq. (22)], so $\tilde{\rho}_0'^{(\text{nsp})} = \tilde{\rho}_0^{(\text{nsp})}$. The identity therefore extends term by term: $G_x |\tilde{\rho}_0^{\text{tot}}\rangle = G_x |\tilde{\rho}_0^{\text{tot}}\rangle - |\psi_0^{\text{NO}}\rangle$, and the full (spectator + nonspectator) NEB cross section is the same in both forms. This equivalence is established here in the reduced single-channel limit, where the optical propagator G_x is common to both forms. Its scope should not be confused with the classic post-prior asymmetry of the standard sum rule: the prior form differs from the post form by the nonorthogonality and interference terms [Eqs. (32) and (33)] regardless of the spectator approximation, and the nonspectator correction $V_{bA} - U_{bA}$, which cancels in $V_{\text{post}} - V_{\text{prior}}$, enters both forms identically and therefore introduces no additional post-prior terms of its own.

IV. RELATION TO THE ICHIMURA-AUSTERN-VINCENT FORMALISM

The spectator approximation can be understood through the Feshbach projection formalism [10,11] in the internal space of b . There are two distinct questions here, and it is useful to keep them separate from the outset. The first is what the spectator approximation removes from the exact composite-fragment problem. The second is how the standard *structureless* IAV observable should be interpreted when it is embedded back into that composite Hilbert space. Defining the projectors $\mathcal{P} = |\phi_0\rangle\langle\phi_0|$ and $\mathcal{Q} = 1 - \mathcal{P}$ onto b 's ground state and excited states, the full scattering state $|\Psi\rangle$ has both $\mathcal{P}$ and $\mathcal{Q}$ components. At the level of the underlying composite-fragment problem, the experiment can in principle distinguish two classes of observables. Detecting b in its ground state probes the $\mathcal{P}$ channel. Detecting excited states of b , or the breakup products of b , probes the $\mathcal{Q}$ sector. The exact formalism developed here computes the $\mathcal{P}$ -channel cross section, namely, the observable in which the detected fragment is projected onto ϕ_0 as in Eq. (8). How the standard structureless IAV should be interpreted relative to that distinction is a separate question, addressed below.

The spectator approximation replaces $V_{bA}(\mathbf{r}_{bA}, \zeta, \xi)$ by an optical potential $U_{bA}(\mathbf{r}_{bA})$ that is independent of b 's internal coordinates ζ and the target coordinates ξ . Because U_{bA} does not depend on ζ , it is diagonal in b 's eigenstates: $\langle\phi_0|U_{bA}|\phi_n\rangle = U_{bA} \delta_{n0}$. This means that the off-diagonal

matrix elements $V_{0n}^{(bA)} = \langle\phi_0|V_{bA}|\phi_n\rangle$ for $n \geq 1$, which connect the $\mathcal{Q}$ components of the projectile wave function to the detected $\mathcal{P}$ state through the b - A interaction, are discarded. The spectator approximation does not, however, eliminate all $\mathcal{Q} \rightarrow \mathcal{P}$ transitions: the binding interaction V_{bx} can still change b 's internal state (since it depends on ζ), and the corresponding matrix elements $\langle\phi_0|V_{bx}|\phi_n\rangle$ are retained. What is lost specifically is the role of V_{bA} in mediating transitions between b 's internal states during the interaction with the target, and the ξ dependence of the diagonal matrix element $V_{00}^{(bA)}$ that couples b 's finite size to target excitations.

The nonspectator generalization developed in this paper retains the full V_{bA} with all its ζ and ξ dependence, including both the diagonal and off-diagonal matrix elements in b 's internal space. The nonspectator source $\rho_0^{(\text{nsp})}$ [Eq. (10)] captures precisely the contributions that the spectator approximation misses. Expanding the projectile wave function in b 's eigenstates, $\varphi_a(\mathbf{r}_{bx}, \zeta) = \sum_n a_n(\mathbf{r}_{bx}) \phi_n(\zeta)$, the nonspectator source receives contributions from two distinct mechanisms:

$$\begin{aligned} \langle\phi_0|(V_{bA} - U_{bA})|\varphi_a\rangle_\xi &= (V_{00}^{(bA)} - U_{bA}) a_0(\mathbf{r}_{bx}) \\ &+ \sum_{n \geq 1} V_{0n}^{(bA)} a_n(\mathbf{r}_{bx}), \end{aligned} \quad (35)$$

where $V_{0n}^{(bA)} = \langle\phi_0|V_{bA}|\phi_n\rangle_\xi$. The first term is the diagonal correction: b remains in its ground state ($\mathcal{P}$ space), but the true ground-state interaction $V_{00}^{(bA)}(\mathbf{r}_{bA}, \xi)$ differs from the optical potential $U_{bA}(\mathbf{r}_{bA})$ because it retains the ξ dependence and finite-size effects averaged out in U_{bA} . The second term is the $\mathcal{Q}$ -space contribution: b is found in an excited component ϕ_n (with $n \geq 1$) within the projectile a , and the off-diagonal matrix element $V_{0n}^{(bA)}$ mediates the transition $\mathcal{Q} \rightarrow \mathcal{P}$ during the interaction with the target. This term represents the coherent return from $\mathcal{Q}$ space to $\mathcal{P}$ space that the experiment detects but the spectator approximation discards. In the spectator limit $V_{bA} \rightarrow U_{bA}$, both terms vanish: $V_{00}^{(bA)} \rightarrow U_{bA}$ and $V_{0n}^{(bA)} \rightarrow U_{bA} \delta_{n0}$, recovering $\rho_0^{(\text{nsp})} = 0$ and the IAV sum rule.

What observable does the structureless IAV represent? Any identification of the structureless IAV with a composite-fragment observable is necessarily approximate, requiring at minimum a closure approximation [$G_{x,n} \approx G_x$, with the channel-dependent propagator $G_{x,n}$ defined in Eq. (36) below], identical exit-channel distorted waves and detector acceptance for all internal states n , and the structureless reduction $(\varphi_a, V_{bx}) \rightarrow (\varphi_a^{\text{sl}}, V_{bx}^{\text{eff}})$. With these caveats stated at the outset, I now show that the structureless IAV most naturally corresponds to the *total* inclusive cross section summed over all of b 's internal states, rather than the state-resolved cross section for any individual ϕ_n .

The post-form master formula [Eq. (8)] and its prior-form counterpart [which in the reduced single-channel limit becomes Eq. (27)] describe the state-resolved observable in which the detected fragment is projected onto ϕ_0 . The standard IAV formalism, by contrast, does not carry an explicit projection onto any internal state of b , because b has been made structureless from the start.

The distinction between the state-resolved and total observables is most transparent in the language of projection

operators in b 's internal space. Working within the reduced single-channel limit where the explicit target dependence has been suppressed, define the unprojected source $\mathcal{S}(\mathbf{r}_x, \zeta) = \langle \chi_b^{(-)} | V_{\text{post}} \chi_a^{(+)} \varphi_a \rangle_{\mathbf{r}_{bb}}$, which retains b 's internal coordinate ζ through $V_{bx}(\mathbf{r}_{bx}, \zeta)$, $V_{bA}(\mathbf{r}_{bA}, \zeta)$, and $\varphi_a(\mathbf{r}_{bx}, \zeta)$; the target coordinate ξ no longer appears in V_{bA} here because, in this reduced single-channel limit, V_{bA} is built from nucleon optical potentials without explicit ξ dependence (Sec. II). The state-resolved NEB cross section for detecting b in internal state ϕ_n projects this source with $\mathcal{P}_n = |\phi_n\rangle\langle\phi_n|$:

$$\sigma_n^{\text{NEB}} = -\frac{(2\pi)^4}{\pi v_a} \langle \mathcal{S} | \mathcal{P}_n \otimes G_{x,n}^\dagger W_{x,n} G_{x,n} | \mathcal{S} \rangle, \quad (36)$$

where $G_{x,n} = (E_{x,n}^+ - E_A - K_x - U_{x,n})^{-1}$ is the $x + A$ optical propagator and $W_{x,n} = \text{Im } U_{x,n}$ the corresponding absorptive potential, both evaluated at the channel energy $E_{x,n} = E - E_b - \varepsilon_n$ appropriate to detecting b in its internal state ϕ_n at the same fixed kinetic energy E_b that defines the double-differential cross section. For $n = 0$ they reduce to the ground-state quantities G_x [defined following Eq. (11)] and $W_x = \text{Im } U_x$ [Eq. (17)] introduced above, with $U_{x,0} \equiv U_x$ the optical potential evaluated at the ground-state channel energy $E_{x,0}$. If the internal excitation energies $\varepsilon_n - \varepsilon_0$ are small compared to $E_{x,0}$, one may approximate $G_{x,n} \approx G_x$ for all relevant n (closure approximation). The total NEB cross section then becomes, by completeness $\sum_n \mathcal{P}_n = \mathbf{1}_\zeta$,

$$\sigma_{\text{total}}^{\text{NEB}} = \sum_n \sigma_n^{\text{NEB}} \approx -\frac{(2\pi)^4}{\pi v_a} \langle \mathcal{S} | \mathbf{1}_\zeta \otimes G_x^\dagger W_x G_x | \mathcal{S} \rangle. \quad (37)$$

The projection $\mathcal{P}_0 = |\phi_0\rangle\langle\phi_0|$ selects the ground-state channel σ_0^{NEB} ; the identity $\mathbf{1}_\zeta$ sums over all channels, giving $\sigma_{\text{total}}^{\text{NEB}}$.

The source-level projection $\mathcal{P}_0$ versus $\mathbf{1}_\zeta$ has a direct counterpart at the Hamiltonian level through the Feshbach formalism. The state-resolved cross section σ_0^{NEB} corresponds to working in the $\mathcal{P}$ -projected space. In the Feshbach framework, the effective $\mathcal{P}$ -space Hamiltonian includes a polarization potential from virtual excitations to $\mathcal{Q}$ space [10,11],

$$H_{\text{eff}} = \mathcal{P}H\mathcal{P} + \mathcal{P}H\mathcal{Q}(E - \mathcal{Q}H\mathcal{Q})^{-1}\mathcal{Q}H\mathcal{P}, \quad (38)$$

which accounts for the fact that, even when b is detected in its ground state, it may have been virtually excited to $\mathcal{Q}$ space during the reaction. The total cross section $\sigma_{\text{total}}^{\text{NEB}}$, by contrast, sums over all $\mathcal{P}$ and $\mathcal{Q}$ final states and therefore uses the *full* Hamiltonian H , with no Feshbach reduction in b 's internal $\mathcal{P}$ or $\mathcal{Q}$ space.

The standard structureless IAV has no ζ degree of freedom and no $\mathcal{P}$ or $\mathcal{Q}$ decomposition. In the composite language, it does not impose a projection onto any particular internal state of b . Within the composite framework, this is the analog of using $\mathbf{1}_\zeta = \mathcal{P} + \mathcal{Q}$ (the trace over all internal states) rather than the ground-state projection $\mathcal{P}_0$. The difference between the total and state-resolved observables is the contribution of the real $\mathcal{Q}$ final channels: the cross sections σ_n^{NEB} with $n \geq 1$, which the trace $\mathbf{1}_\zeta$ includes and the ground-state projection excludes. Virtual $\mathcal{Q}$ excursions that return to the detected ground state are, by contrast, part of σ_0^{NEB} itself, entering through the polarization term of H_{eff} . Subject to the closure and structureless-reduction approximations stated above, the

structureless IAV is therefore associated with the total cross section,

$$\sigma_{\text{IAV}}^{\text{NEB}}(\text{structureless } b) \longleftrightarrow \sigma_{\text{total}}^{\text{NEB}} = \sum_n \sigma_n^{\text{NEB}}, \quad (39)$$

rather than the state-resolved cross section σ_0^{NEB} for detecting b in its ground state. Beyond the caveats already stated at the outset (closure, identical exit-channel distorted waves and detector acceptance, structureless reduction), one qualification deserves emphasis: for $n \geq 1$, the experimental observable is the detection of b in an excited bound state or of its breakup products, either of which is a different final-state selection from detecting an intact b in its ground state; in practice, the kinetic energy E_b and angular distribution of b also differ between the ground state and excited or continuum states. Equation (39) should therefore be understood as a conceptual correspondence valid under these approximations, not as an exact identity.

For the deuteron, the closure approximation is qualitatively valid: the only bound state is the ground state ($\varepsilon_0 = -\varepsilon_d = -2.224$ MeV, with ε_d the deuteron binding energy), and the low-lying continuum states that carry most of the spectral weight of φ_a have excitation energies of a few MeV, modest compared to typical reaction energies $E_{x,0} \sim 10\text{--}30$ MeV. The approximation is least controlled near the Coulomb barrier, where $E_{x,0}$ is smallest and W_x varies rapidly with energy; there the channel-dependent propagator $G_{x,n}$ should be retained rather than replaced by G_x .

This correspondence has two distinct implications. At the *formalism* level, the nonspectator generalization provides corrections to the IAV source through $V_{bA} - U_{bA}$, improving the dynamical content of the calculation. At the *observable* level, the present formalism computes σ_0^{NEB} (detecting b in its ground state), whereas the structureless IAV approximates $\sigma_{\text{total}}^{\text{NEB}}$ (summed over all internal states of b). These are in general different observables, and the nonspectator formalism does not merely “correct” the IAV result but accesses a state-resolved quantity that the structureless theory cannot distinguish. The $\mathcal{Q}$ -channel cross sections σ_n^{NEB} ($n \geq 1$), corresponding to detection of b in an excited state or of its breakup products, can be obtained from the same formalism by replacing $\langle\phi_0|$ with $\langle\phi_n|$ in the source function, with the obvious extension to continuum-normalized states or discretized bins for n in the continuum.

The EBU-NEB decomposition of the original IAV applies to σ_{sp} through $\text{Im } G_x = G_x^\dagger W_x G_x + (\text{EB})$. Accordingly, once the full source is considered, both EBU and NEB are organized as in Eqs. (20) and (21). The quantities $\sigma_{\text{sp}}^{\text{EBU}}$ and $\sigma_{\text{sp}}^{\text{NEB}}$ are only the spectator parts of those full observables. For the nonspectator terms, an analogous decomposition exists via the spectral representation of G_{full} but involves inelastic target channels.

Finally, since within the DWBA the nonspectator physics resides entirely in the source [Eq. (8)], a beyond-DWBA treatment replaces the entrance-channel distorted wave $\chi_a^{(+)}\varphi_a$ by a coupled-channel solution such as the CDCC wave function [38], modifying the source while leaving the propagator structure unchanged. The present nonspectator correction to the source applies equally in that framework.

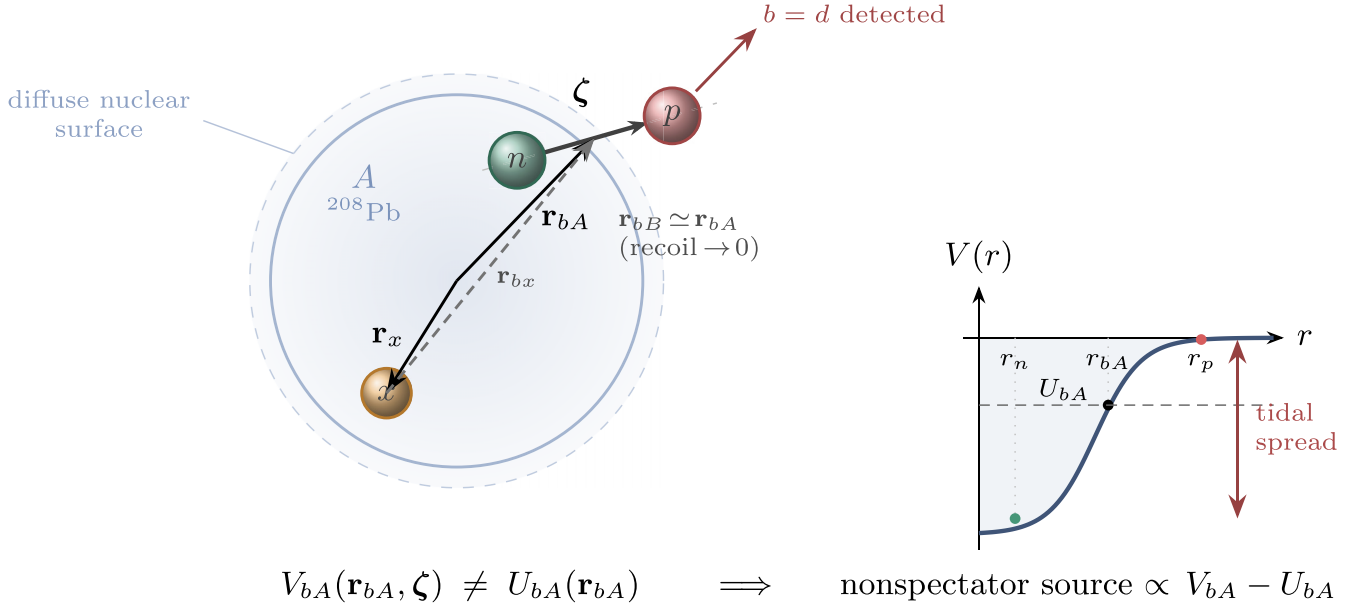


FIG. 1. Schematic of the nonspectator (tidal) mechanism, illustrated for $b = d$ on ^{208}Pb . Left: In the exit channel the detected deuteron $b = p + n$ grazes the diffuse nuclear surface, with center-of-mass position $\mathbf{r}_{bA}$ and internal (p - n) coordinate $\boldsymbol{\zeta}$ drawn as a vector from n to p ; the unobserved fragment x is absorbed by the target. The Jacobi coordinates used in the text are indicated, with $\mathbf{r}_{bB} \simeq \mathbf{r}_{bA}$ up to the residual recoil. Right: The b - A potential $V(r)$ sampled along the line through the constituents. The spectator approximation replaces the true interaction by the optical potential U_{bA} , the potential a structureless (point) b would feel at its center of mass r_{bA} (dashed line). Because the proton (outer, r_p) and neutron (inner, r_n) sit at different radii, they sample V at different depths; this spread is the tidal variation that U_{bA} averages out, and it drives the nonspectator source through the residual operator $V_{bA} - U_{bA}$.

V. TIDAL ESTIMATE FOR THE DEUTERON

To assess the magnitude of the operator $V_{bA} - U_{bA}$ (illustrated in Fig. 1), I consider $b = d$ (deuteron) on ^{208}Pb . For the deuteron I write $\mathbf{r}_d \equiv \mathbf{r}_{bA}$ for the b - A separation introduced in Sec. II. The deuteron-target interaction for a two-body p - n system with internal coordinate $\boldsymbol{\zeta}$ is

$$V_{dA}(\mathbf{r}_d, \boldsymbol{\zeta}, \xi) = V_{pA}(\mathbf{r}_d + \frac{1}{2}\boldsymbol{\zeta}, \xi) + V_{nA}(\mathbf{r}_d - \frac{1}{2}\boldsymbol{\zeta}, \xi). \quad (40)$$

For the estimate below, V_{pA} and V_{nA} are taken as nucleon optical potentials and the explicit ξ dependence is suppressed, as in the reduced single-channel limit of Sec. II. A natural reference potential is the adiabatic folding potential [23,39], defined by averaging the interaction of Eq. (40) over the deuteron ground state,

$$U_d^{\text{fold}}(\mathbf{r}_d) = \int |\phi_d(\boldsymbol{\zeta})|^2 [V_{pA} + V_{nA}] d\boldsymbol{\zeta}. \quad (41)$$

This choice ensures that the diagonal nonspectator correction $\langle \phi_d | (V_{dA} - U_d^{\text{fold}}) | \phi_d \rangle$ vanishes by construction, isolating the off-diagonal (tidal) contributions. The dependence on the choice of reference potential is discussed in Sec. VI.

Expanding V_{dA} in a Taylor series of $\boldsymbol{\zeta}$ about $\mathbf{r}_d$ gives

$$V_{dA} = \Sigma V(\mathbf{r}_d) + \boldsymbol{\zeta} \cdot \nabla_{\mathbf{r}_d} \delta V + \frac{1}{8} \zeta_i \zeta_j \partial_i \partial_j \Sigma V + \dots, \quad (42)$$

where $\Sigma V = V_{pA} + V_{nA}$ is the isoscalar sum and $\delta V = (V_{pA} - V_{nA})/2$ is the isovector difference, both evaluated at $\mathbf{r}_d$; the coefficients follow from the $\pm \boldsymbol{\zeta}/2$ arguments in Eq. (40). The zeroth-order (monopole) term $\Sigma V(\mathbf{r}_d)$ is the zero-range limit

of the folding potential; the true folding potential [Eq. (41)] includes finite-size corrections: for an s -wave deuteron,

$$U_d^{\text{fold}} = \Sigma V + \frac{\langle \zeta^2 \rangle}{24} \nabla^2 \Sigma V + \dots, \quad (43)$$

where $\langle \zeta^2 \rangle = \int |\phi_d|^2 \zeta^2 d\boldsymbol{\zeta}$ and the dipole average $\langle \boldsymbol{\zeta} \rangle = 0$ by parity. The nonspectator operator is therefore

$$V_{dA} - U_d^{\text{fold}} = \boldsymbol{\zeta} \cdot \nabla \delta V + \frac{1}{8} \left(\zeta_i \zeta_j - \frac{\delta_{ij}}{3} \langle \zeta^2 \rangle \right) \partial_i \partial_j \Sigma V + \dots \quad (44)$$

The first (dipole) term couples the deuteron's $E1$ response to the gradient of the isovector potential δV ; it enters the nonspectator source through off-diagonal matrix elements $\langle \phi_0 | \boldsymbol{\zeta} | \phi_n \rangle$. The second (quadrupole) term couples the $E2$ response to the curvature of the isoscalar potential ΣV . Each multipole provides a systematic truncation for the coupled-channel evaluation of the nonspectator terms.

Since the diagonal matrix element vanishes with this choice of U_{bA} , a useful diagnostic is the rms fluctuation $\Delta V_{\text{tidal}}(r_d) = \sqrt{\langle \phi_d | (V_{dA} - U_d^{\text{fold}})^2 | \phi_d \rangle}$. Because the diagonal element vanishes by the folding choice, ΔV_{tidal} is, by closure, the root-sum-square of all off-diagonal couplings $\langle \phi_0 | (V_{dA} - U_d^{\text{fold}}) | \phi_n \rangle$; it is thus the aggregate strength that actually feeds the nonspectator source, not merely a diagonal expectation value. For a back-of-the-envelope estimate, consider first the dipole term. The isovector nuclear potential for ^{208}Pb has a typical strength of $|V_p - V_n|/2 \approx 5\text{--}8$ MeV (predominantly the symmetry or Lane potential), which changes over a surface

TABLE I. Comparison of the standard IAV formalism (spectator approximation) with the nonspectator generalization derived in this work. Post-prior equivalence is listed for the reduced single-channel limit, with the nonorthogonality and interference terms completing the prior form.

Quantity	IAV	Nonspectator
b structure	Structureless	Composite
V_{post}	$V_{bx} + U_{bA} - U_{bB}$	$V_{bx} + V_{bA} - U_{bB}$
Internal-state treatment	No explicit internal-state projection	Explicit $\langle \phi_0 $ projection
Observable (conceptual)	$\sigma_{\text{total}}^{\text{NEB}}$ (under closure, structureless reduction)	σ_0^{NEB}
Post-prior	Yes	Yes (reduced single-channel limit)

diffuseness of ~ 0.7 fm, giving an isovector gradient $|\nabla \delta V| \sim 10$ MeV/fm. For the Coulomb contribution, the gradient at the nuclear surface ($r \approx 7$ fm) is $|\nabla V_C|/2 \approx Ze^2/(2r^2) \approx 1.5$ MeV/fm, which is subdominant. With the deuteron's rms p - n separation $\sqrt{\langle \zeta^2 \rangle} \approx 3.9$ fm, the dipole tidal correction is of order $|\nabla \delta V| \times \zeta_{\text{rms}}/\sqrt{3} \sim 10 \times 2.25 \approx 22$ MeV, where the factor $1/\sqrt{3}$ arises from the angular average of $\zeta \cdot \hat{\nabla}$ over the s -wave deuteron. While this is smaller than the depth of V_{dA} itself, it is an order of magnitude larger than the deuteron binding energy ($\epsilon_d = 2.224$ MeV). The quadrupole contribution involves the traceless combination $(\zeta_i \zeta_j - \delta_{ij} \langle \zeta^2 \rangle/3)$ coupled to $\partial_i \partial_j \Sigma V$. For a Woods-Saxon potential with depth $V_0 \sim 50$ MeV, radius $R \approx 7$ fm, and diffuseness $a \approx 0.65$ fm, the isoscalar Laplacian at the nuclear surface is dominated by the $2V'/r$ contribution (the radial second derivative V'' vanishes at $r = R$ for a Woods-Saxon shape), giving $|\nabla^2 \Sigma V| \sim V_0/(aR) \sim 11$ MeV/fm². With $\langle \zeta^2 \rangle \approx 15$ fm², the quadrupole scale is $\langle \zeta^2 \rangle |\nabla^2 \Sigma V|/8 \sim 21$ MeV, comparable to the dipole estimate. This uses the scalar Laplacian as a dimensional scale; the traceless quadrupole operator in fact contracts with the anisotropy $V'' - V'/r$ of the Hessian rather than its trace $V'' + 2V'/r$, so the value is an order-of-magnitude bound rather than the precise quadrupole matrix element. The relevant quantity for the nonspectator source is the off-diagonal transition matrix element $\langle \phi_0 | Q_{ij} | \phi_n \rangle$, with $Q_{ij} = \zeta_i \zeta_j - \delta_{ij} \langle \zeta^2 \rangle/3$, which is reduced by the rapid falloff of the deuteron wave function at large ζ . These estimates indicate that the nonspectator operator is not a small perturbation at the nuclear surface. Whether this operator-level scale translates into a significant difference between σ_0^{NEB} and $\sigma_{\text{total}}^{\text{NEB}}$ requires numerical evaluation of the source integrals.

VI. DISCUSSION

The main results of this work are summarized in Table I, which compares the standard IAV formalism with the nonspectator generalization. The central finding is that, within the DWBA, all nonspectator effects are encoded in the operator $V_{bA} - U_{bA}$ acting in the source function, while the propagator remains unchanged at each level of description: the full $x + A$ resolvent in the exact result, and its optical reduction G_x in the reduced single-channel limit. Because this operator modifies the source before the final-state projection, it can in principle change both the elastic-breakup and nonelastic-breakup components of the observed intact- b cross section, even though the practical IAV machinery is primarily aimed at NEB. This operator has a physical interpretation as the tidal correction

across b 's internal extent (Fig. 1): the individual constituents of b experience different nuclear potentials at their respective positions, and this spatial variation is averaged out in the optical potential U_{bA} . For compact fragments such as the α particle ($\epsilon_\alpha = 28.3$ MeV, $r_{\text{rms}} \approx 1.5$ fm), the tidal correction is small and the spectator approximation is accurate. For the deuteron ($\epsilon_d = 2.224$ MeV, rms p - n separation ≈ 3.9 fm), the tidal estimate of Sec. V puts the nonspectator operator at roughly 20 MeV at the nuclear surface, an order of magnitude above the deuteron binding energy.

The regime where nonspectator effects are expected to be most important includes low-energy reactions ($E_a \lesssim 30$ MeV/nucleon), where the projectile spends significant time in the nuclear field; heavy targets (large A), where the nuclear potential varies strongly over the deuteron's spatial extent; and reactions near the Coulomb barrier, where stripping and transfer processes compete with direct breakup. At high energies ($E_a > 100$ MeV/nucleon), the eikonal approximation becomes valid, the deuteron traverses the nuclear field rapidly, and nonspectator effects are expected to be suppressed [40]. A related effect has been identified at intermediate energies in nucleon-knockout reactions [27], where a double Feshbach projection of the three-body Hamiltonian reveals that the standard additive model for composite projectiles misses induced interactions from excluded projectile configurations, leading to systematic overestimates of the stripping cross section and apparent spectroscopic quenching.

For practical implementation, it is again useful to distinguish the simplified and fully microscopic levels. In the simplified scheme where V_{bA} is built from nucleon optical potentials and does not depend explicitly on ξ , the calculation remains close to existing IAV machinery, but one should distinguish the post and prior representations. In post form, one replaces the standard source $\tilde{\rho}_0$ by $\tilde{\rho}_0^{\text{tot}} = \tilde{\rho}_0 + \tilde{\rho}_0^{(\text{ns})}$ in Eq. (24). In prior form, which is what practical codes usually implement, the analogous extension is to add the reduced nonspectator source $\tilde{\rho}_0^{(ns)}$ to the UT source, the NO term being pure spectator, while retaining the same optical propagation. The genuinely new step in either representation is the evaluation of the reduced nonspectator source, which requires integration over b 's internal coordinate ζ with the coordinate-dependent operator $V_{bA}(\mathbf{r}_{bA}, \zeta) - U_{bA}(\mathbf{r}_{bA})$. For the deuteron, $V_{bA} = V_{pA}(\mathbf{r}_{bA} + \zeta/2) + V_{nA}(\mathbf{r}_{bA} - \zeta/2)$ is not separable in $\mathbf{r}_{bA}$ and ζ , so this step introduces additional partial-wave couplings and requires a new numerical module. Several features make the computation genuinely difficult. First, the six-dimensional integral over $\mathbf{r}_{bB}$ [three-dimensional (3D)] and ζ

(3D) must be performed for each value of $\mathbf{r}_x$; the nonseparability prevents factorization into independent radial integrals. Second, the multipole expansion of $V_{bA} - U_{bA}$ (Sec. V) converges slowly for a spatially extended fragment such as the deuteron ($\zeta_{\text{rms}} \approx 3.9$ fm, several times the surface diffuseness over which the potential varies), so many partial waves are needed. Third, the projectile wave function $\varphi_a(\mathbf{r}_{bx}, \zeta)$ is itself a multicoordinate object: for reactions where a is a three-body system (e.g., ${}^6\text{Li} = \alpha + d = \alpha + n + p$), computing φ_a already requires a three-body calculation. A perturbative evaluation of the leading (dipole) correction may be the most feasible first step, since the $E1$ matrix elements $\langle \phi_0 | \zeta | \phi_n \rangle$ can be extracted from CDCC bin states and the resulting source has a structure similar to existing coupled-channel IAV calculations. The full nonperturbative evaluation, including all multipoles, remains an open computational challenge.

The fully microscopic problem is qualitatively harder still. Once V_{bA} carries explicit ξ dependence and target excitations are retained, the single-channel Green's function G_x must be replaced by the coupled-channel object $[\mathbf{G}_x]_{\alpha\beta}$, and the calculation moves well beyond standard IAV infrastructure. The analytical estimates of Sec. V are therefore not a substitute for a numerical calculation, but serve to identify the physical regime where nonspectator effects matter and the multipoles that dominate.

Within the reduced single-channel limit, the decomposition into spectator, cross, and nonspectator terms depends on the choice of the reference potential U_{bA} . The total source $\tilde{\rho}_0^{\text{tot}} = \tilde{\rho}_0 + \tilde{\rho}_0^{(\text{nsp})}$ and the total cross section are independent of U_{bA} (since the U_{bA} terms cancel between the two contributions), but the individual spectator and nonspectator terms are not. A natural choice is the adiabatic folding potential $U_{bA} = U_{bA}^{\text{fold}}$ [Eq. (41)], which makes the diagonal correction vanish, $\langle \phi_0 | (V_{bA} - U_{bA}) | \phi_0 \rangle = 0$, so that the nonspectator source is driven entirely by off-diagonal ($\mathcal{Q} \rightarrow \mathcal{P}$) transitions. If instead an empirical b - A optical potential is used for U_{bA} , one must be aware that such potentials are typically fitted to elastic scattering data and may already absorb some $\mathcal{P} \leftrightarrow \mathcal{Q}$ polarization effects through their imaginary part and effective real potential [8,41]. In that case, the nonspectator correction $V_{bA} - U_{bA}$ should be understood as the residual beyond what the empirical optical model already accounts for, and care must be taken to avoid double counting.

If $V_{bA} - U_{bA}$ is treated as a perturbation on the spectator result, the NEB cross section can be expanded as

$$\sigma^{\text{NEB}} \approx \sigma_{\text{sp}}^{\text{NEB}} + \sigma_{\text{cross}}^{\text{NEB,(1)}} + \mathcal{O}[(V_{bA} - U_{bA})^2], \quad (45)$$

where $\sigma_{\text{sp}}^{\text{NEB}}$ [Eq. (19)] is the composite spectator NEB cross section and the first-order correction $\sigma_{\text{cross}}^{\text{NEB,(1)}}$ is the NEB part of the cross term, obtained by applying the $G_x^\dagger W_x G_x$ projection of Eq. (17) to the cross term [Eq. (15)]. For a given choice of

U_{bA} , this provides a diagnostic of the spectator approximation: if $\sigma_{\text{cross}}^{\text{NEB,(1)}}$ is small compared to $\sigma_{\text{sp}}^{\text{NEB}}$, the spectator approximation is justified for that reaction relative to the chosen reference potential.

VII. SUMMARY

I have derived a generalization of the IAV inclusive breakup formalism that removes the spectator approximation and provides state-resolved inclusive cross sections for composite fragment b . At the exact DWBA level, the generalized sum rule involves the full $x + A$ resolvent $(E_{x,0}^+ - H_{xA})^{-1}$, while the familiar single-channel IAV structure is recovered only after an additional reduction that suppresses the explicit target dependence of V_{bA} . Under a closure approximation, the standard IAV formalism with structureless b is associated with the total inclusive cross section summed over all of b 's internal states, without distinguishing whether the detected fragment is in its ground state or has been excited or broken up. The present formalism introduces the projection onto b 's ground state explicitly, yielding the cross section σ_0^{NEB} for detecting b intact, which is a distinct observable from the total $\sigma_{\text{total}}^{\text{NEB}}$ to which the IAV formalism corresponds under closure and structureless-reduction approximations. Within the DWBA, all nonspectator effects enter through the source function via the operator $V_{bA} - U_{bA}$, while the propagator is unchanged at each level of description. Because the source is modified before the EBU or NEB projection is taken, nonspectator dynamics can in principle alter both the elastic and nonelastic breakup components of the observed intact- b signal. Post-prior equivalence is preserved: V_{bA} cancels exactly in $V_{\text{post}} - V_{\text{prior}}$, and the equivalence is verified explicitly in the reduced single-channel limit. A tidal estimate for $b = d$ on ${}^{208}\text{Pb}$ shows that the nonspectator operator is not a small perturbation at the nuclear surface; whether this operator-level scale produces a significant difference between σ_0^{NEB} and $\sigma_{\text{total}}^{\text{NEB}}$ requires numerical evaluation of the source integrals.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this article. Requests for further information or data should be sent to the author.

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