Assessing continuum channel importance in continuum-discretized coupled-channels via dynamic polarization potential decomposition

Jin Lei [1,2,*] and Hao Liu [1]
[1] School of Physics Science and Engineering, Tongji University, Shanghai 200092, China
[2] Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, Guangdong Province, China

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A recurring question in continuum-discretized coupled-channels calculations is which continuum channels carry the breakup coupling, both to interpret the reaction and to decide which channels a model space can safely omit. The standard answer is bin deletion: remove a channel, re-solve the coupled equations, and read the change in the elastic S matrix. We show that this cannot isolate an individual channel's contribution. Deleting a channel forces the surviving model space to reorganize, with the neighboring bins rerouting through the off-diagonal Green's function, so the recorded change mixes the channel's own effect with the readjustment of all the others. Working instead from the channel-resolved Feshbach dynamic polarization potential (DPP), whose full-coupling Green's function is a fixed reference shared by every channel, we define an exclusion that removes one channel while holding that reference fixed, returning its contribution to the intact system. The two operations disagree on which continuum bins matter most, for $d + {}^{58}$Ni at 21.6 MeV even sending the intrinsically leading bin to last place under deletion. The DPP further separates each channel's action into a direct path, virtual excitation and return, and a bridge path, relayed through neighboring bins, a split deletion cannot make; it shows the bins act on the elastic channel mainly as bridges, robustly across angular momentum, and that it is this bridge coupling that reorganizes under deletion. A deletion-based channel importance is therefore best read as the truncated calculation's sensitivity to a channel's removal, not as the channel's intrinsic coupling strength.

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I. INTRODUCTION

In nuclear reactions involving weakly bound projectiles, coupling to breakup continuum channels can profoundly alter elastic scattering and reaction cross sections [1]. The continuum-discretized coupled-channels (CDCC) method [2–4] handles this by discretizing the projectile continuum into energy bins and solving the resulting coupled equations. A persistent practical question is which continuum channels contribute most to the breakup coupling effect, a question that is essential for understanding breakup-modified elastic scattering, guiding model-space truncation, and interpreting converged results.

In CDCC, each continuum bin is a square-integrable wave packet labeled by the relative orbital angular momentum l of the projectile fragments and a narrow excitation-energy interval; together with the projectile ground state, a chosen set of such bins spans the model space in which the dynamics are evaluated. The projectile-target couplings among the states of this space define the coupled radial equations whose solution yields the elastic and breakup S matrices at each total angular momentum J, and any observable is converged by enlarging the model space (raising the partial-wave cutoff $l_{\max}$ and the bin-energy cutoff $E_{\max}$) until the result stabilizes.

The standard approach has been bin deletion: remove a channel from the coupled equations, re-solve, and measure the change in the elastic S matrix [5]. This technique has been widely applied to compare resonant and nonresonant breakup states [5–8], to quantify continuum-continuum coupling effects [9], to identify dominant breakup components [10,11], to map coupling form factors [12], and to study model-space convergence [13]. However, because deletion removes a channel from the coupled set entirely, it also severs the bridge pathways that run through that channel, and the surviving channels propagate through a reorganized Green's function: the measured response reflects not only the deleted channel's own contribution but also the readjustment of the surviving basis. What it produces is thus a property of the truncated calculation, not an intrinsic property of the channel.

The Feshbach projection formalism [14–16] provides a channel-importance measure that avoids this complication. The effect of all eliminated channels on elastic scattering is encoded in the dynamic polarization potential (DPP), a nonlocal effective interaction that has been extracted from coupled-channel S matrices by inversion [17,18]. The breakup-induced DPP was first characterized by Sakuragi, Yahiro, and Kamimura [19], who found it repulsive and weakly absorptive, qualitatively unlike a collective-coupling DPP; later work decomposed it by continuum subspace, computing the DPP from coupling to resonant versus nonresonant states [6–8]. The present work carries this DPP decomposition down to the individual bin and, within each bin, separates the

*Contact author: jinl@tongji.edu.cn

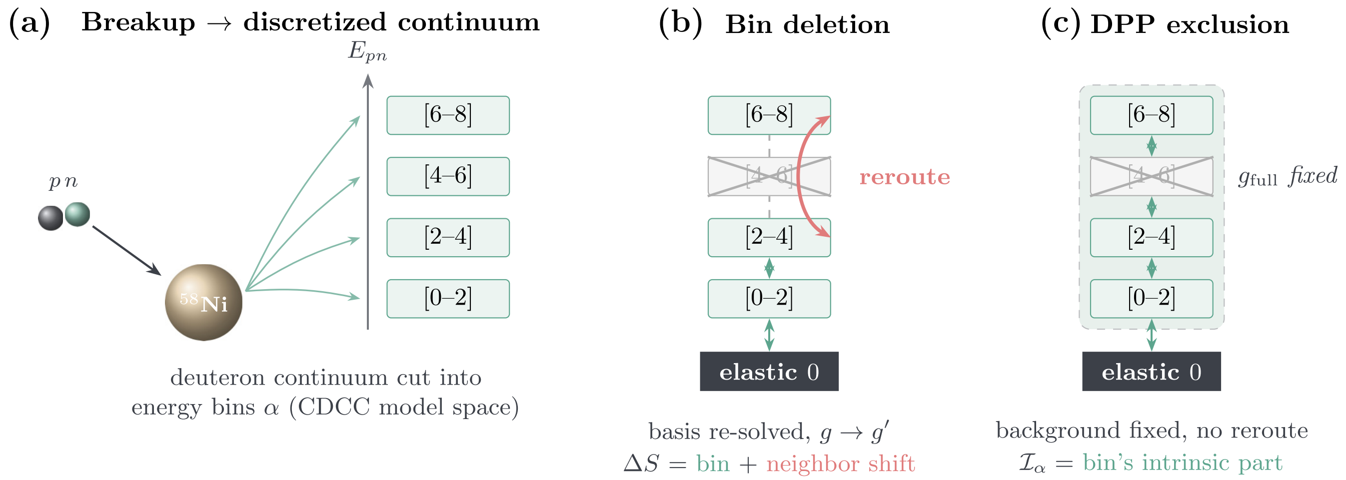

FIG. 1. Two ways to assess a continuum channel's importance in CDCC. (a) The projectile continuum is discretized into energy bins α; the bins, together with the projectile ground state, span the model space coupled to the elastic channel. (b) Bin deletion removes a channel from the coupled set and re-solves, so the Green's function is rebuilt ($g \to g'$, the reduced-space $g^{(-\alpha)}$ of Sec. II) and the surviving bins reroute through the off-diagonal coupling; the recorded ΔS mixes the deleted bin's own contribution with the readjustment of its neighbors. (c) The DPP exclusion removes the same channel while holding the full-coupling Green's function g_{full} fixed as a shared reference, leaving the other bins untouched and returning the channel's intrinsic contribution $\mathcal{I}_\alpha$.

direct pathway from the bridge pathway. The full-coupling, channel-resolved DPP recently computed in our companion paper [20] enables a direct channel-by-channel decomposition: the DPP is a bilinear sum over channel pairs, each term linking the elastic channel out to one continuum channel and back in from another through the full-coupling Green's function, so that every term is labeled by a pair of channels. A channel can then be removed by dropping the terms that carry its label, with that Green's function held fixed, whereas deletion re-solves a reduced coupled system. Each channel is thereby assessed against an identical dynamical background, a measure of its contribution to the intact system rather than of the truncated system's response to its removal (Fig. 1). In this work we use this construction to show that deletion and the intrinsic measure rank the continuum bins differently: at $J = 8$ deletion ranks the [0.32–0.39] fm^{-1} bin highest while the intrinsic measure ranks [0.39–0.47] fm^{-1}, a reordering we trace, both algebraically and numerically, to the reorganization of the surviving model space. The decomposition further resolves each channel's contribution into a direct (self-coupling) part and a bridge (interchannel coherence) part, a separation unique to the DPP route, and reveals that the continuum bins act on the elastic channel predominantly through bridges; this bridge dominance, robust across angular momentum, is exactly the part of the dynamics that reorganizes under deletion.

The argument unfolds in three steps. We first cast the channel-importance question in terms of the channel-resolved DPP, define the intrinsic importance measure, and split it into direct and bridge parts (Sec. II). We then apply it to $d + {}^{58}$Ni, where the measure and standard bin deletion rank the bins differently, a gap we trace to the reorganization of the surviving model space, and where the direct/bridge decomposition shows the coupling is bridge dominated (Sec. III). We close by drawing out what this means for channel-importance diagnostics in CDCC (Sec. IV).

II. DYNAMIC POLARIZATION POTENTIAL AND ITS DECOMPOSITION

A continuum bin can act on elastic scattering in two physically distinct ways: directly, by carrying the projectile into the bin and back to its ground state, or as a bridge, by relaying the flux through the coupling network of the other bins. The Feshbach formalism [14,15] makes this separation exact. Projecting onto the elastic channel ($\mathcal{P}$) and onto the eliminated continuum bins ($\mathcal{Q} = 1 - \mathcal{P}$), the elastic-channel effective Hamiltonian acquires the dynamic polarization potential

$$\Delta U(R, R') = \sum_{\gamma,\gamma'} U_{0\gamma}(R)\, g_{\gamma\gamma'}(R, R')\, U_{\gamma'0}(R'), \tag{1}$$

where R is the projectile-target radial coordinate, $U_{0\gamma}$ couples the elastic channel to continuum bin γ (labeled by the orbital angular momentum l and the excitation-energy interval), and $g_{\gamma\gamma'}(R, R')$ are the channel matrix elements of the full-coupling Green's function $G_J^{(+)}(R, R')$, which retains all continuum-continuum couplings [20,21]. Here $G_J^{(+)} = (E - H^J_{\mathcal{Q}\mathcal{Q}})^{-1}$ is the resolvent of the $\mathcal{Q}$-space Hamiltonian at total angular momentum J, so Eq. (1) is the exact Feshbach polarization potential for the elastic channel, with no weak-coupling or perturbative approximation. The off-diagonal elements $g_{\gamma\gamma'}$ ($\gamma \neq \gamma'$) carry the coherence between eliminated bins that builds the bridge paths, and are absent in weak-coupling treatments [9].

Grouping the double sum by channel participation decomposes the DPP into direct and bridge contributions for each channel α:

$$\Delta U^{(\alpha)} = D_\alpha + B_\alpha, \quad D_\alpha = U_{0\alpha}\, g_{\alpha\alpha}\, U_{\alpha 0},$$
$$B_\alpha = \sum_{\gamma\neq\alpha} (U_{0\alpha}\, g_{\alpha\gamma}\, U_{\gamma 0} + U_{0\gamma}\, g_{\gamma\alpha}\, U_{\alpha 0}), \tag{2}$$

with $\Delta U = \sum_\alpha D_\alpha + \frac{1}{2}\sum_\alpha B_\alpha$, the factor $\frac{1}{2}$ correcting the double counting of each off-diagonal bridge pair, which appears once in B_α and again in B_γ. The direct term D_α represents virtual excitation elastic $\rightarrow \alpha \rightarrow$ elastic via the diagonal propagator $g_{\alpha\alpha}$; the bridge term B_α captures multistate paths mediated by the off-diagonal $g_{\alpha\gamma'}$, so a bin with weak direct coupling can still contribute through coherence with its neighbors.

To weigh a single bin α, we drop it from the sum in Eq. (1) while leaving $g_{\gamma\gamma'}$ untouched. This is an algebraic operation on the already-computed DPP, not a fresh coupled-channel solve: the Green's function is built once and reused, so removing α changes nothing about how any other bin propagates. We measure the effect on the elastic S matrix through

$$\mathcal{I}_\alpha = \frac{|S_{\rm full} - S_{{\rm no}\,\alpha}|}{|S_{\rm full}|}, \tag{3}$$

where $S_{\rm full}$ and $S_{{\rm no}\,\alpha}$ are the elastic S-matrix elements at fixed total angular momentum J, and $\Delta S_\alpha \equiv S_{\rm full} - S_{{\rm no}\,\alpha}$. Because every bin is weighed against the same fixed continuum, the importances $\mathcal{I}_\alpha$ are directly comparable from one bin to the next; this is the intrinsic measure we use throughout. They are not additive, since S responds nonlinearly to ΔU: what the construction provides is not additivity but a common, unmoving reference. We call $\mathcal{I}_\alpha$ *intrinsic* in this precise sense, that every channel is weighed against an identical, unmoving dynamical background; it remains a property of the channel within the fully coupled system, not of an isolated channel, which has no meaning once the continuum is coupled.

This exclusion has a transparent meaning inside CDCC itself. The $\mathcal{Q}$-space Hamiltonian, and hence g, depends only on the continuum-continuum couplings and never on the elastic couplings $U_{0\gamma}$, so setting $U_{0\alpha} = U_{\alpha 0} = 0$ in the coupled-channels Hamiltonian and re-solving leaves g pointwise unchanged and returns the same $S_{{\rm no}\,\alpha}$. The operation closes the elastic doorway of bin α while leaving it fully embedded in the network of continuum-continuum couplings through the untouched $U_{\gamma\alpha}$ ($\gamma \neq 0$): what it isolates is the channel's effect with the rest of the network held fixed. A direct CDCC re-solve with $U_{0\alpha} = U_{\alpha 0} = 0$ reproduces the fixed-g exclusion importance to within 0.4 percentage points across the midcontinuum bins at $J = 8$, rising for the two highest bins near the channel-closure threshold to the level of the reconstruction floor, the numerical accuracy with which the DPP rebuilds the full elastic S matrix (Sec. III). The equality is exact at the operator level, since g is independent of the elastic couplings $U_{0\alpha}$; the residual is numerical, of the same origin as that reconstruction floor, not a breakdown of the identity. Codes such as FRESCO [22] specify couplings by form factor and partition rather than by individual matrix elements, so this re-solve, like the DPP construction itself, operates at the matrix-element level; of the two the DPP route is the more economical, since one full-coupling Green's-function evaluation assesses every channel at once. The decomposition into direct and bridge terms, Eq. (2), is intrinsic to the DPP and cannot be recovered from S matrix differences alone: it names the physical pathway by which each channel acts, one-step excitation versus coherence with its neighbors.

Standard bin deletion does something different and more drastic. It takes α out of the $\mathcal{Q}$ space altogether and solves the coupled equations again in the smaller space that is left. The two operations share one part and differ by one part, and the difference can be written down exactly. This is the comparison the rest of the paper turns on, so we set it out term by term. Exclusion, which drops α from the sum in Eq. (1) while holding g fixed, removes only the terms that contain α,

$$\delta U_\alpha^{\rm excl} = \Delta U^{(\alpha)} = D_\alpha + B_\alpha, \tag{4}$$

exactly the channel contribution $\Delta U^{(\alpha)}$ already written in Eq. (2). At fixed g the change produced by excluding α, $\delta U_\alpha^{\rm excl}$, coincides with the contribution assigned to α by the decomposition, $\Delta U^{(\alpha)}$; the exclusion operation and the decomposition thus yield the same quantity. The factor $\frac{1}{2}$ in $\Delta U = \sum_\alpha D_\alpha + \frac{1}{2}\sum_\alpha B_\alpha$ does not enter the single-channel exclusion. It corrects a double counting present only in the summation over all channels: each off-diagonal bridge pair contributes to both B_α and B_γ, so $\sum_\alpha B_\alpha$ enumerates every pair twice. The exclusion of a single channel involves no such summation. Because the bridge connecting α and γ cannot exist in the absence of α, excluding α omits both of the terms in which it appears, those carrying $g_{\alpha\gamma}$ and $g_{\gamma\alpha}$, from the sum at once, so the full B_α is removed rather than $\frac{1}{2}B_\alpha$. Deletion removes these same terms, but it does one more thing. The reduced system is itself a coupled-channels problem, so its elastic-channel effective potential has the same exact Feshbach form as Eq. (1), only built on the Green's function $g^{(-\alpha)}$ of the one-bin-smaller continuum space: with the continuum space reduced, the propagator itself changes, from g to $g^{(-\alpha)}$. The total change recorded by deletion is therefore

$$\begin{aligned} \delta U_\alpha^{\rm del} &= \delta U_\alpha^{\rm excl} + R_\alpha, \\ R_\alpha &= \sum_{\gamma,\gamma' \neq \alpha} U_{0\gamma}\big(g_{\gamma\gamma'} - g^{(-\alpha)}_{\gamma\gamma'}\big)U_{\gamma' 0}. \end{aligned} \tag{5}$$

Equation (5) is an exact identity, not an approximation. The extra piece R_α, which we call the *reorganization term*, is built only from the surviving bins, since its sum runs over $\gamma, \gamma' \neq \alpha$ and never touches α itself. It measures how much the surviving bins' own contribution to the elastic channel shifts once α is pulled out of the propagator g they all share, and it vanishes when α decouples from the surviving bins, being generically nonzero otherwise. In short, exclusion records $\Delta U^{(\alpha)}$, the contribution of bin α against a fixed background, while deletion records $\Delta U^{(\alpha)} + R_\alpha$, that same contribution plus the readjustment of every other bin to a background that has changed.

The reorganization term has a clear physical meaning. Deleting a bin does more than erase that bin's own contribution: it forces every surviving bin to propagate through a Green's function that has lost one of its coupling partners, so the surviving bins shift as well. The magnitude of this shift can be made explicit. Splitting the $\mathcal{Q}$-space Hamiltonian into a block for α and a block for the survivors and inverting, the survivors' Green's function comes out as the reduced-space one, $g^{(-\alpha)}$, plus a correction built from the flux that leaves the survivors, passes through α, and comes back,

$$g_{\gamma\gamma'} - g^{(-\alpha)}_{\gamma\gamma'} = [g^{(-\alpha)}\,\Sigma_\alpha\, g^{(-\alpha)}]_{\gamma\gamma'} + \mathcal{O}(\Sigma_\alpha^2),$$
$$\Sigma_\alpha = V_{r\alpha}\,\frac{1}{E - H_{\alpha\alpha}}\,V_{\alpha r}, \tag{6}$$

where Σ_α is exactly that round trip through α, and $V_{\alpha r}$ are the couplings of α to the survivors r. These are the same couplings that build the bridge term B_α (through $g_{\alpha r} = g_{\alpha\alpha}\,V_{\alpha r}\,g^{(-\alpha)}$), so, at leading order, the reorganization is driven by the very matrix elements that make a channel a bridge, and the bridge-dominated channels are the ones most sensitive to deletion. This correspondence holds exactly at the first order shown; the higher-order terms in Eq. (6) involve longer paths through α, and the first-order term alone already accounts for the reorganization-bridge correspondence found below. Its sign is not fixed: for complex, multichannel potentials, R_α can add to or subtract from the intrinsic shift, so deletion neither systematically over- nor understates a channel's contribution. What deletion records, $\delta U_\alpha^{\rm excl} + R_\alpha$, inseparably mixes the channel's own effect with the readjustment of all the others. The two operations thus answer different questions: the intrinsic contribution of a channel, and the sensitivity of the truncated calculation to its removal.

III. RESULTS

We test the construction on $d + {}^{58}$Ni elastic scattering at $E_{\rm lab} = 21.6$ MeV, asking whether bin deletion and the intrinsic measure agree on which continuum bins carry the breakup coupling. We run the comparison in the $l = 2$ sector, with the ground state and the $l = 0$ and $l = 1$ couplings held fixed as the dynamical background. The $l = 2$ quadrupole is the discriminating choice. The $l = 1$ dipole is the longest-range, dominant breakup mode, but its strength is concentrated in the low-energy bins just above threshold, a steep hierarchy in which the leading bins are not in serious doubt; the $l = 0$ monopole carries no off-diagonal Coulomb coupling and contributes weakly. Neither sector, then, poses the question this work turns on: whether the two diagnostics rank the bins differently. The eight $l = 2$ bins instead carry comparable nuclear and Coulomb-quadrupole strength spread from threshold to the open-channel limit, where the ranking is genuinely contested and the choice of diagnostic can matter. The two measures do not rank the $l = 2$ bins in the same order: their rankings reorganize relative to each other, most strongly at the grazing partial wave $J = 8$, on which we concentrate. The deuteron continuum is discretized into bins of equal width in the n-p relative momentum within each l sector, a standard CDCC prescription: six bins for $l = 0$ and $l = 1$ (relative energy 0 to 12 MeV, $\Delta k = 0.087$ fm^{-1}, k to 0.54 fm^{-1}), and, for $l = 2$, which carries the importance analysis, a ladder of eight bins extended nearly to the open-channel limit (relative energy 0 to 16 MeV, $\Delta k = 0.076$ fm^{-1}, k to 0.62 fm^{-1}) so that the interpreted bins are free of truncation. The n-p interaction is a central Gaussian of depth 72.15 MeV and range 1.484 fm, reproducing the 2.224-MeV deuteron binding energy [3]; the nucleon-target interactions are Koning-Delaroche optical potentials [23] at half the deuteron laboratory energy; the coupled equations are solved by the Lagrange-mesh R-matrix method (200 basis functions, 60-fm matching radius), with the potentials tabulated on a 0.05-fm radial grid. The DPP exclusion and the CDCC deletion use the same Hamiltonian, bins, and numerical parameters and differ only in how a channel is removed, so any disagreement between them reflects the difference between the two questions, not between two calculations. As a control on the decomposition, solving the one-channel elastic equation with the bare (folded) potential plus the DPP of Eq. (1) reproduces the full CDCC elastic S matrix to within 3.1%, 1.5%, and 0.8% at $J = 6$, 8, and 10, in each case below the interpreted single-bin importances at that J (the lowest two bins, which sit at this reconstruction floor, are not interpreted). The reconstruction is tightest for a compact continuum and loosens as the ladder is carried toward the channel-closure threshold. In its present form the channel-resolved DPP is evaluated over the open channels only, the closed-channel Whittaker continuation used by standard deletion has not yet been implemented, and its reconstruction degrades when the highest bin is pushed against that closure threshold, which sets the eight-bin ladder used here; this is a limitation of the construction, not of the comparison. Both diagnostics are evaluated on the identical open-channel basis, so no part of their disagreement can come from seeing different model spaces, and the inequivalence itself [Eq. (5)] is projection algebra that holds on any basis; the individual importances, like any CDCC quantity, do depend on the basis. The operator identity $U_{\rm CDCC} = U_{\rm bare} + U_{\rm DPP}$, with $U_{\rm bare}$ being the bare elastic (folded) potential and $U_{\rm DPP} = \Delta U$ being the polarization potential of Eq. (1), is established and shown to be exact in the companion work [20,21], so this residual is of numerical, not physical, origin. The agreement confirms that the channel-resolved decomposition faithfully represents the CDCC dynamics.

The breakup-induced DPP has so far been available only in local, approximate form: as an angular-momentum-averaged local potential in the founding work [19] and the foundational CDCC review [3], and resolved by continuum subspace, resonant versus nonresonant, in the local-equivalent prescription [6–8]. The exact, fully nonlocal DPP, the rigorous Feshbach polarization potential evaluated with the full-coupling Green's function, was first constructed in our companion work [20,21]. What is new here is to resolve that exact DPP down to the individual continuum bin and, within each bin, to separate the direct pathway from the bridge pathway.

The intrinsic importance of the $l = 2$ bins (Table I) shows a clear hierarchy: it is negligible for the two near-threshold bins ([0.02–0.09] and [0.09–0.17] fm^{-1}, at the reconstruction floor and not interpreted) and is largest in the upper continuum, peaking at the [0.39–0.47] fm^{-1} bin (18.5%) and then falling toward the open-channel edge. The hierarchy is not a simple function of bin energy, but it mirrors the strength of the couplings themselves: the doorway form factors $U_{0\alpha}(R)$ are concentrated at the nuclear surface and are strongest for the midcontinuum bins, peaking at [0.39–0.47] fm^{-1}, the same bin the importance ranks first at $J = 8$ and $J = 10$. The importance peak also migrates with J (Table II), from [0.24–0.32] fm^{-1} at $J = 6$ to [0.39–0.47] fm^{-1} at $J = 8$ and $J = 10$. The static form factors cannot carry this migration,

TABLE I. Single-bin importance $\mathcal{I}_\alpha$ (%) for the $l = 2$ continuum bins at $J = 8$. $\mathcal{I}^{\mathrm{DPP}}$ is the DPP channel exclusion (net of direct and bridge); $\mathcal{I}^D$ and $\mathcal{I}^B$ are the direct-only and bridge-only exclusions, which do not sum to $\mathcal{I}^{\mathrm{DPP}}$ because S depends nonlinearly on the potential; and $\mathcal{I}^{\mathrm{del}}$ is the standard CDCC bin deletion. Ranks of $\mathcal{I}^{\mathrm{DPP}}$ and $\mathcal{I}^{\mathrm{del}}$ (in parentheses, 1 = largest, each column ranked independently) are taken over the six interpreted bins; the two near-threshold bins, [0.02–0.09] and [0.09–0.17], lie at the DPP reconstruction floor ($\lesssim$1.5%), are not physically interpreted, and are excluded from the ranking. The bins span the $l = 2$ continuum up to $k = 0.62$ fm^{-1}, just below the channel-closure threshold. The two orderings disagree sharply: the intrinsic measure ranks the [0.39–0.47] fm^{-1} bin first while deletion ranks it last.

Bin k (fm^{-1})	$\mathcal{I}^{\mathrm{DPP}}$	$\mathcal{I}^D$	$\mathcal{I}^B$	$\mathcal{I}^{\mathrm{del}}$
[0.02–0.09]	0.0	0.0	0.1	0.0
[0.09–0.17]	1.5	0.9	2.3	1.8
[0.17–0.24]	8.6 (5)	6.7	12.7	9.7 (4)
[0.24–0.32]	11.6 (3)	15.3	21.1	18.7 (2)
[0.32–0.39]	9.5 (4)	15.9	22.5	20.8 (1)
[0.39–0.47]	18.5 (1)	12.0	28.0	1.7 (6)
[0.47–0.55]	16.4 (2)	5.9	19.4	17.8 (3)
[0.55–0.62]	4.1 (6)	3.3	4.1	4.4 (5)

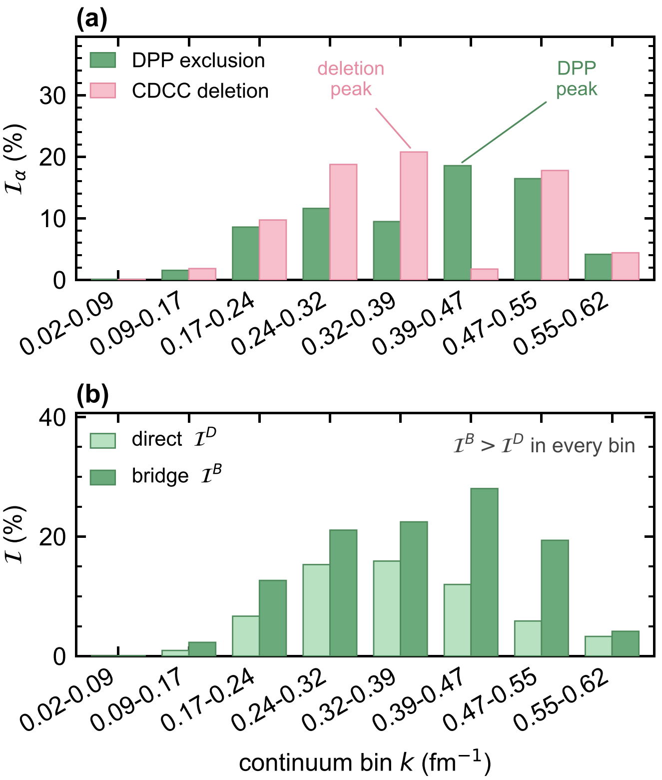

FIG. 2. Single-bin importance $\mathcal{I}_\alpha = |\Delta S_\alpha|/|S_{\mathrm{full}}|$ for $d + {}^{58}$Ni at 21.6 MeV, $J = 8$, and $l = 2$. (a) DPP channel exclusion versus standard CDCC bin deletion: the two rank the bins differently, deletion peaking at [0.32–0.39] fm^{-1} and the intrinsic measure at [0.39–0.47] fm^{-1}. (b) Direct-only ($\mathcal{I}^D$) versus bridge-only ($\mathcal{I}^B$) exclusion: the bridge contribution exceeds the direct one for all interpreted bins and does so decisively across the midcontinuum, the signature of bridge-dominated coupling.

being independent of J and, if anything, shorter ranged for the higher bins; the migration is a property of the propagation, of how the rising centrifugal barrier reweights the contributions of the bins. The overall $l = 2$ importance falls steeply with J, as the rising barrier screens the surface region where the coupling is concentrated.

The direct/bridge decomposition [Eq. (2)] explains why a single coupling-strength number is the wrong picture. In every interpreted bin, the bridge exclusion exceeds the direct one ($\mathcal{I}^B > \mathcal{I}^D$, Table I; at the highest open bin the margin lies within the reconstruction floor), so each bin acts on the elastic channel mainly by relaying flux through its neighbors rather than by coupling back on its own. $\mathcal{I}^D$ and $\mathcal{I}^B$ switch off, respectively, a bin's diagonal term and all off-diagonal terms involving it, one at a time; they are separate single-pathway removals, not additive parts of $\mathcal{I}^{\mathrm{DPP}}$. The [0.32–0.39] fm^{-1} bin makes the point sharply: its net importance ($\mathcal{I}^{\mathrm{DPP}} = 9.5\%$) is smaller than either single-pathway value ($\mathcal{I}^D = 15.9\%$, $\mathcal{I}^B = 22.5\%$). The direct-only and bridge-only removals shift S by $\Delta S^D = (-0.027, -0.047)$ and $\Delta S^B = (+0.049, +0.059)$, two large, nearly antiparallel vectors in the complex plane (relative phase 170°), while removing the channel altogether leaves only the small net $\Delta S = (+0.027, +0.017)$. The modest net importance is thus a coherent near-cancellation between strong direct and bridge couplings acting in nearly opposite directions, not a weak underlying coupling; because S is nonlinear in ΔU, these two single-pathway shifts need not sum to the net. That a channel acts on the elastic scattering mainly through this back-and-forth, multistep coupling rather than through a one-step return is in keeping with earlier dynamic polarization potentials for weakly bound systems, in which the elastic cross section is shaped by the coupling back and forth between the bound and continuum states [24].

Standard deletion, shown alongside the intrinsic measure in Table I and Fig. 2, gives a different ordering: it ranks the [0.32–0.39] fm^{-1} bin most important and [0.24–0.32] fm^{-1} second, whereas the intrinsic measure ranks [0.39–0.47] fm^{-1} first and [0.47–0.55] fm^{-1} second. The disagreement is not marginal: the bin the intrinsic measure ranks first, [0.39–0.47] fm^{-1}, deletion ranks last of the six interpreted bins, and the two orderings stay inequivalent at $J = 6$ as well, agreeing on the leading bin only at $J = 10$ (Table II); they differ most at $J = 8$. Nor is deletion a uniform rescaling of the intrinsic measure: at $J = 8$ the ratio $\mathcal{I}^{\mathrm{DPP}}/\mathcal{I}^{\mathrm{del}}$ runs from 0.5 at [0.32–0.39] fm^{-1} to above 10 at [0.39–0.47] fm^{-1}, where the bin the intrinsic measure ranks first is nearly invisible to deletion, so deletion overstates some channels and understates others.

This is the behavior predicted by Eq. (5): deletion records $\delta U_\alpha^{\mathrm{excl}} + R_\alpha$, and the reorganization R_α, built from the same bridge couplings, shifts each channel by a different, sign-indefinite amount. The data bear this out. Define the reorganization signature $|\Delta S_\alpha^{\mathrm{del}} - \Delta S_\alpha^{\mathrm{excl}}|$, the complex gap between the deletion and exclusion shifts. Across the interpreted $l = 2$ bins it peaks together with the bridge importance $\mathcal{I}_\alpha^B$ at the high-bridge [0.39–0.47] fm^{-1} bin; this is exactly the

TABLE II. Single-bin $l = 2$ importance (%) from DPP exclusion ($\mathcal{I}^{\rm DPP}$) and CDCC deletion ($\mathcal{I}^{\rm del}$) at $J = 6,\ 8,$ and 10, with ranks in parentheses (1 = largest, each column ranked independently). Ranks are over the same six interpreted bins at every J; the two near-threshold bins ([0.02–0.09], [0.09–0.17]), which sit at the reconstruction floor ($\lesssim$1.5%) at $J = 8$, are excluded throughout for uniformity. The bins extend up to $k = 0.62$ fm^{-1}, just below the channel-closure threshold. The two orderings are inequivalent at $J = 6$ and $J = 8$, agreeing on the leading bin only at $J = 10$, with the strongest reordering at $J = 8$.

	$J = 6$		$J = 8$		$J = 10$	
Bin k (fm^{-1})	DPP	del	DPP	del	DPP	del
[0.02–0.09]	0.0	0.1	0.0	0.0	0.0	0.0
[0.09–0.17]	3.9	2.5	1.5	1.8	0.6	0.7
[0.17–0.24]	24.8 (4)	9.8 (6)	8.6 (5)	9.7 (4)	2.2 (5)	3.2 (5)
[0.24–0.32]	40.9 (1)	16.7 (5)	11.6 (3)	18.7 (2)	2.6 (3)	4.2 (3)
[0.32–0.39]	39.1 (2)	36.3 (1)	9.5 (4)	20.8 (1)	4.1 (2)	3.5 (4)
[0.39–0.47]	19.3 (5)	20.0 (3)	18.5 (1)	1.7 (6)	4.4 (1)	7.7 (1)
[0.47–0.55]	38.8 (3)	28.6 (2)	16.4 (2)	17.8 (3)	2.4 (4)	4.5 (2)
[0.55–0.62]	17.5 (6)	19.8 (4)	4.1 (6)	4.4 (5)	1.3 (6)	1.2 (6)

bin on whose rank deletion and the intrinsic measure disagree most, the intrinsic measure ranking it first and deletion last, so the disagreement is driven by the bridge-built reorganization. Deletion thus reports how sensitive the truncated calculation is to a channel's removal, not the channel's intrinsic contribution.

The same picture holds away from $J = 8$. At $J = 6$ and $J = 10$ (Fig. 3) the bridge dominance persists: $\mathcal{I}^B > \mathcal{I}^D$ for every interpreted $l = 2$ bin at every angular momentum, with margins that exceed the reconstruction floor in every mid-continuum bin, by factors from about 1.5 to more than 10; only at the highest open bin at $J = 6$ and 8 does the margin fall within the floor, where the ordering is consistent with, rather than established by, the calculation. The conclusion that the continuum couples to the elastic channel mainly through interchannel coherence therefore rests on no single bin or angular momentum. The deletion-versus-intrinsic ranking, by contrast, is angular-momentum dependent (Table II): the two orderings differ at $J = 6$ and $J = 8$, most strongly at $J = 8$, and agree on the leading bin only at $J = 10$. In every case the per-bin ratio $\mathcal{I}^{\rm DPP}/\mathcal{I}^{\rm del}$ spreads on both sides of unity. Deletion therefore provides neither the intrinsic ranking nor a fixed rescaling of the intrinsic magnitude at any J, consistent with the reorganization term R_α of Eq. (5) being channel dependent and sign indefinite.

A channel-importance analysis is only meaningful if the continuum ladder is itself converged, free of an artificial accumulation of flux in the highest bin when that bin has no higher partner to couple to. The $l = 2$ ladder used throughout approaches the open-channel limit: eight bins out to $k = 0.62$ fm^{-1}, just below the channel-closure threshold ($E_{\rm c.m.} - B_d \approx$ 18.7 MeV, $k \approx 0.67$ fm^{-1}, where $B_d = 2.224$ MeV is the deuteron binding energy paid to dissociate it) at which the $d + {}^{58}$Ni breakup channels close. The elastic S matrix is converged on this basis: enlarging the $l = 2$ ladder from six to seven to eight bins (which also refines the momentum grid) shifts it by only 2.8% and then 1.1%, and the per-bin importance turns over at high k, the highest open bin ($k = 0.55$–0.62 fm^{-1}) being small in both diagnostics ($\mathcal{I}^{\rm DPP} = 4.1\%$, $\mathcal{I}^{\rm del} =$ 4.4%; Table I and Fig. 2). There is thus no flux pile-up at the

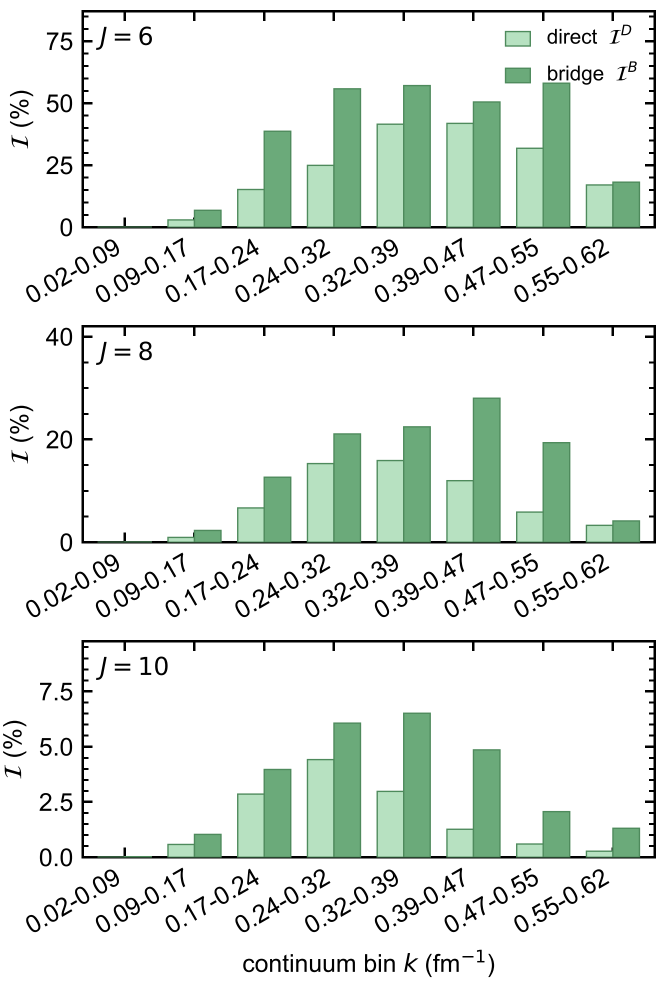

FIG. 3. Angular-momentum robustness of bridge dominance. Direct-only ($\mathcal{I}^D$, light) and bridge-only ($\mathcal{I}^B$, dark) exclusions for the $l = 2$ bins at $J = 6,\ 8,$ and 10. The bridge contribution exceeds the direct one in all interpreted bins at every angular momentum and does so decisively across the midcontinuum, so the bridge-dominated character of the continuum coupling is not specific to $J = 8$.

truncation edge; the importance is largest in the mid-k region, where the doorway form factors are strongest, and falls off smoothly toward the open-channel boundary. Extending the $l = 0$ and $l = 1$ ladders to the same limit alongside $l = 2$ leaves the $l = 2$ per-bin importance essentially unchanged: no single-bin importance moves by more than 2 percentage points, the deletion ordering is identical in the two bases, and the DPP ordering changes only by an interchange of its fourth- and fifth-ranked bins, so the profile is a property of the continuum dynamics, not of an unbalanced truncation.

IV. DISCUSSION AND OUTLOOK

The picture that emerges is coherent and practical. Bin deletion, the standard tool for judging continuum channel importance, answers a well-defined question, but not the one it is usually assumed to answer. Removing a channel sets off a reorganization of the surviving model space [Eq. (5)], in which the neighboring bins, coupled to the removed channel through the same matrix elements that build the bridge term, reroute around it. What deletion records is therefore an inseparable mixture of the channel's own contribution and this reorganization, and it ranks the bins differently from the intrinsic measure, the per-bin importance ratio at $J = 8$ running from 0.5 to above 10. Deletion-based channel importances in the literature [6,7,9] are, on this reading, best interpreted as a measure of how sensitive a truncated calculation is to a channel's removal, not as the channel's intrinsic contribution to the breakup coupling. The complementarity runs both ways: that removal sensitivity is itself the relevant quantity for the practical question of which channels a finite model space may safely omit, so for model-space truncation the deletion ranking is the appropriate diagnostic, whereas the intrinsic measure, which holds the background fixed, ranks channels by physical contribution and is not a guide to truncation efficiency.

Two results follow. The first is conceptual: the inequivalence of DPP exclusion and coupled-channel deletion follows from the Feshbach projection algebra [Eqs. (5) and (6)] and holds for any projected coupled-channels system, with the reorganization term R_α governed by the bridge couplings and, in general, sign indefinite. The $l = 2$ results shown here are therefore representative; by the same algebra the inequivalence is expected to apply to the $l = 0$ and $l = 1$ sectors as well, though we do not quantify it there. The second is physical: the direct/bridge decomposition, available only from the DPP, shows that the continuum bins couple to the elastic channel predominantly as bridges, by relaying flux through neighboring bins rather than by coupling back on their own, a feature robust across angular momentum. Both require the channel-resolved DPP, hence the full-coupling Green's function $G_J^{(+)}(R, R')$ of the companion work [21], which standard CDCC codes such as FRESCO [22] do not generate. Where that machinery is unavailable, a deletion-based importance from a standard CDCC code should still be read as a ranking of truncation sensitivity, not of intrinsic coupling strength; the direct versus bridge character cannot be recovered from deletion at all, since deleting a channel removes both contributions together while reorganizing the survivors in one inseparable step.

The $d + {}^{58}$Ni case studied here is a representative, well-controlled test. We expect, though do not demonstrate here, the effects it isolates to be strongest in halo systems such as ^{6}He, ^{11}Li, and ^{11}Be, where the spatially extended projectile is likely to enhance the off-diagonal Green's function couplings: there the continuum coupling should be even more thoroughly bridge dominated, the reorganization under deletion correspondingly larger, and the gap between what a channel intrinsically contributes and what its removal appears to cost at its widest. These are precisely the reactions in which the choice of which continuum channels to retain most strongly shapes the predicted observables.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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